

This postprint was originally published by Elsevier as: Zilker, V. (2022). **Stronger attentional biases can be linked to higher reward rate in preferential choice.** *Cognition, 225,* Article 105095. <u>https://doi.org/10.1016/j.cognition.2022.105095</u>

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Stronger attentional biases can be linked to higher reward rate in preferential choice

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Abstract

When attention is biased to a particular option during information search preceding preferential choice, this option is often more likely to be chosen—even if its value is objectively lower than that of the alternative. This article demonstrates that although attentional biases—even to lower-valued options—may reduce accuracy (the tendency to choose the highest-valued option), they can increase reward rate (the amount of reward obtained per unit of time invested in the choice). To achieve a higher reward rate it is often preferable to choose a lower-valued option quickly rather than spend time trying to identify the highest-valued option. Attentional biases are typically associated with faster choices, and in terms of reward rate, this reduction in response time can often compensate for the accompanying decrease in accuracy. This relationship between attention, response time, and reward rate is modulated by features of the choice environment and by individual differences in choice boundaries and in the attentional amplification of evidence accumulation. These patterns are predicted theoretically by the attentional drift-diffusion model (aDDM). A reanalysis of empirical data from several eye-tracking studies shows that these predicted patterns also hold empirically across various domains of preferential choice (riskless and risky choice, options of monetary rewards and of food items). It may therefore often be beneficial for decision makers to allocate their attention in a biased manner—that is, to deliberately ignore information on some options—in order to reduce the time cost of choice and thereby achieve a higher reward rete.

Article Info

Keywords: Attention, Attentional bias, Accuracy, Reward rate, Response time, Attentional drift-diffusion model

Attention allocation is closely linked to preferences. For instance, when choosing between two options that are each associated with a value, people are more likely to favor the option they look at longer (e. g., Armel, Beaumel, & Rangel, 2008; Cavanagh, Wiecki, Kochar, & Frank, 2014; Fiedler & Glöckner, 2012; Konovalov & Krajbich, 2016; Krajbich, Armel, & Rangel, 2010; Krajbich, Lu, Camerer, & Rangel, 2012; Krajbich & Rangel, 2011; Shimojo, Simion, Shimojo, & Scheier, 2003; Stewart, Hermens, & Matthews, 2016). The attentional drift-diffusion model (aDDM; Krajbich et al., 2010; Krajbich & Rangel, 2011) accounts for this finding with a sequential sampling process in which the drift rate is a function of attention such that evidence tends to accumulate faster for the predominantly attended option. Hence, an option is more likely to be chosen the more attention it receives (relative to the other option), even if it has an objectively lower value than the other option. From the normative stance of maximizing accuracy (i.e., aiming to choose the option with the higher value in each problem) the behavioral consequences of attentional biases, formalized in the aDDM, seem suboptimal: If attention happens to be biased towards the lower-valued option rather than allocated evenly, a systematic reduction in the tendency to choose the higher-valued option is predicted. Even if attention is allocated randomly with respect to the options' values, choices driven by attention become increasingly random and therefore less accurate (Thomas, Molter, Krajbich, Heekeren, & Mohr, 2019).¹

Accuracy is a normative benchmark that is commonly used to evaluate the quality of decisions in experiments (e.g., Pachur, Mata, & Hertwig, 2017; Zilker, Hertwig, & Pachur, 2020). This is in line with neoclassical economic theory, which posits that rational agents should choose the option with the highest expected value or expected utility

¹ Throughout this manuscript the term "attentional bias" is used to refer to attention being allocated unevenly between the available options—that is, one option being inspected for a longer amount of time than the other.

(Bernoulli, 1954). However, holding agents to the normative standard of maximizing accuracy ignores a performance metric that is important in many natural ecologies: the time investment associated with a choice. Work on bounded rationality highlights that simple, fast strategies that function with limited time and information can often fare better than strict accuracy maximization, especially when cognitive and computational resources are limited (e.g., Gigerenzer et al., 1999; Simon, 1956, 1990, 1997). Moreover, speed–accuracy trade-offs, which have been studied in many subdisciplines of psychology (e.g., Payne, Bettman, & Johnson, 1988; Reed, 1973; Salthouse, 1979), suggest that strategies that are less accurate may often be faster. This also seems to hold for attentional biases, which do not only affect which option is chosen, but also how long it takes to arrive at a choice. By assuming that evidence accumulation to the unattended option is discounted (formally, $\theta < 1$, see details below), the aDDM predicts that choice is typically faster when attention is more strongly biased towards one of the options. This also holds empirically (Smith & Krajbich, 2018). Although attentional biases may reduce the tendency to choose the option with the objectively higher value, they may also save time. But how can one tell if this reduction in time cost justifies the associated decrease in accuracy?

A possible answer is provided by one of the classical theories of efficient foraging, the marginal value theorem (Charnov, 1976). This framework posits that organisms should forage for resources in a way that maximizes the reward rate—that is, the reward obtained per unit of time invested. Because the reward rate quantifies trade-offs between rewards obtained and the time costs of decision making, comparing strategies in terms of reward rate makes it possible to assess whether the higher amount of rewards expected under a given strategy justifies its higher time investment. Thereby, reward rate maximization offers an alternative normative framework to accuracy maximization—one that takes into account the time cost of making a choice. This framework has been successfully applied to various animal species (e.g., Cassini, Kacelnik, & Segura, 1990; Cowie, 1977) and even plants (McNickle & Cahill, 2009). A crucial insight when trying to maximize reward rate is that the time cost associated with trying to identify the best among the available options (i.e., maximizing accuracy—for instance, by deliberately ignoring information in order to be faster—can sometimes achieve a higher reward rate, and thus be more efficient. For instance, when foraging for nectar among flowers, bees that tend to be fast but inaccurate achieve a higher nectar collection rate than do slow, accurate bees (Burns, 2005; Chittka, Dyer, Bock, & Dornhaus, 2003).

This rationale can be applied to preferential choice as well; here, the reward rate can be conceptualized as the value of the chosen option (i.e., the obtained reward) divided by the response time. When considering a series of choices rather than a single one, the expected reward rate is given by the expected reward divided by the associated expected response time. If one considers the reward rate as a normative benchmark, obtaining a smaller reward quickly can be preferable to investing more time to obtain a (potentially only slightly) larger reward—even though accuracy maximization would prescribe the opposite. For illustration, consider a choice between two options, A and B, with values $V_A = 10$ and $V_B = 9$, respectively. If it takes a decision maker 5 seconds to identify the higher-valued option with 90% accuracy, the expected reward rate is $(0.9 \cdot 10 + 0.1 \cdot 9)/5=1.98$. An alternative strategy may only take 4 seconds to implement, but achieve only 70% accuracy.² The expected reward rate in that case is $(0.7 \cdot 10 + 0.3 \cdot 9)/4=2.425$ —higher than the first strategy. That is, the reduced time cost of the faster strategy more than compensates for its lower accuracy.³

In this light, the observation that attentional biases can affect both choice behavior and the time invested in a choice raises an interesting possibility: Attentional biases in preferential choice may be linked to a reduced tendency to choose the higher-valued option, but in terms of reward rate this reduction in obtained rewards may be compensated for by the accompanying reduction in response time. In other words, a decision maker may achieve a higher reward rate when relying on biased rather than balanced attention allocation. Conversely, while allocating attention evenly may help them obtain the larger reward, this benefit may be outweighed by the associated increase in time cost when considering the reward rate. Based on this reasoning, this article investigates the normative consequences of attentional biases in terms of the normative benchmarks of accuracy and reward rate. It first shows, through a series of simulations using the aDDM, that attentional biases can be detrimental in terms of accuracy while also being beneficial in terms of reward rate, and delimits the conditions under which this is the case. It then reanalyzes data from four previously published eye-tracking studies in different domains of preferential choice (Fiedler & Glöckner, 2012; Krajbich et al., 2010; Smith & Krajbich, 2018; Stewart et al., 2016) to test whether human decision makers also tend to achieve a higher reward rate when displaying stronger attentional biases before they make a choice. Finally, the empirical behavior is modeled in the aDDM to test how well the theoretically predicted patterns match the empirical observations.

1. The aDDM

In choice problems offering two options A and B with values V_A and V_B , respectively, the aDDM assumes that evidence for each option accumulates over time. At t = 0 the evidence for option A, DV_A (DV: decision variable), and the evidence for option B, DV_B , are initialized as $DV_A(t = 0) = 0$ and $DV_B(t = 0) = 0$. On each subsequent step t (where t = 1 ms) on which option A is attended to, DV_A and DV_B are updated according to

| $DV_A(t) = DV_A(t-1) + d \cdot V_A + \zeta(t)$ | |
|--|-----|
| $DV_B(t) = DV_B(t-1) + d \cdot \theta \cdot V_B + \zeta(t).$ | (1) |

On each time step t where option B is attended to, DV_A and DV_B are updated according to

| $DV_A(t) = DV_A(t-1) + d \cdot \theta \cdot V_A + \zeta(t)$ | |
|---|-----|
| $DV_B(t) = DV_B(t-1) + d \cdot V_B + \zeta(t).$ | (2) |

The parameter $0 < \theta < 1$ captures that on time steps on which option A is attended to, evidence for option B accumulates at a lower rate than on time steps on which option B is attended to, and vice versa. For instance, if $\theta = 0.5$, evidence for an option accumulates at twice the rate when it is attended to compared to when it is not attended to. Lower values of this parameter indicate that attention amplifies the accumulation of evidence in favor of the attended option more strongly. *d* is a scaling parameter. On each time step independent Gaussian samples $\zeta(t)$ from $\mathcal{N} \sim (0, \sigma)$ are drawn and added, rendering the process noisy. Moreover, the difference between the options in evidence

$$RDV(t) = DV_A(t) - DV_B(t)$$

is computed, and the absolute value abs(RDV(t)) is compared to the

(3)

² In this example, the response times and accuracies associated with each strategy can be conceived of as derived from a formal model—which, for simplicity, is not yet further specified. For instance, one could assume that the two example strategies constitute aDDM-processes with two distinct sets of parameters.

³ The benefit of investing additional time depends critically on the reward structure of the environment. For instance, if we assume that the options in the example differ more strongly in their values, with $V_A = 10$ and $V_B = 0$, the expected reward rate is $(0.9 \cdot 10 + 0.1 \cdot 0)/5 = 1.8$ for the first strategy and $(0.7 \cdot 10 + 0.3 \cdot 0)/4 = 1.75$ for the second (given the same response times). Since more can be gained from choosing the higher-valued option in this case, investing more time is justified in terms of both reward rate and accuracy.

decision boundary. Assuming that the upper and lower decision boundary are set at a distance of ± 1 from the starting point, a choice is made once $abs(RDV(t)) \ge 1$. If RDV(t) is positive (negative) at the time of choice, this indicates that the upper (lower) boundary is reached and option A (B) is chosen. The total number of time steps between the onset of accumulation and choice, n_{steps} , constitutes the response time. Dividing the value of the chosen option by the time taken to arrive at a choice yields the reward rate RR for a given response. That is, the reward rate achieved by a given response is $RR = V_A/n_{steps}$ if option A is chosen and $RR = V_B/n_{steps}$ if option B is chosen.

2. Simulation study

A simulation study was conducted to illustrate the normative consequences of attentional biases in terms of accuracy and reward rate in an example set of choice problems and using an example set of aDDM parameters.

2.1. Choice problems

A set of simple choice problems, each consisting of two options, A and B, was constructed. A total of 100 settings for the value of option A, V_A , were independently uniformly sampled from the range 1 to 6.6. Each V_A was paired with five different variants of option B. Their values V_B were obtained by multiplying the respective V_A by 1.1, 1.2, 1.3, 1.4, and 1.5. These five ratios between the options' values can be understood as five different levels of choice difficulty (assuming that it is easier to identify the higher-valued option if the options' values differ more strongly). This procedure results in problems with values ranging between 1 and 10, inspired by the range of values employed in a classical study on the aDDM by Krajbich et al. (2010). Option A was the lower- valued option in each choice problem, and both the absolute values of A and B as well as the differences between them varied across choice problems. This procedure yielded a total set of 500 choice problems. Appendix A reports additional simulations based on choice problems with negative outcomes.

2.2. Differences in attention allocation

When simulating choice processes in the aDDM, it is necessary to determine which option the model attends to on each time step. This process was simulated by independent draws from a Bernoulli distribution on each time step. The binary outcomes of these draws corresponded to the two options: A draw of "1" indicated that the higher-valued option was fixated on a given time step, and a draw of "0" indicated that the lower-valued option was fixated on a given time step. This probability of attending to the higher-valued option in the choice problem (option *B*) on each time step. This probability, $Patt_{highV}$, determines both the direction and severity of attentional bias. Attention is biased towards the higher-valued option if $Patt_{highV} > 0.5$ and towards the lower-valued option if $Patt_{highV} < 0.5$. The severity of attentional biases (to either option) can be calculated as $att. bias = abs(Patt_{highV} - 0.5)$. Note that att. bias ranges from 0 to 0.5, where a value of 0 indicates that attention is allocated evenly between both options, and a value of 0.5 indicates that the option towards which attention is biased to exclusively. The assumptions about the direction and severity of attentional biases were systematically varied in independent runs of the simulation. In separate runs of the simulation, $Patt_{highV}$ was varied within [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0], thus covering scenarios in which attention was biased to the higher-valued option, allocated evenly, and biased to the lower-valued option. This made it possible to systematically explore how the strength and direction of attentional biases affected accuracy, response time, and reward rate.

2.3. Data generation

Besides the strength and direction of attentional biases, the central parameter governing the attentional amplification of evidence accumulation in the aDDM, θ , was also systematically varied. Specifically, θ was varied within [0, 0.25, 0.5, 0.75, 1.0], where lower values indicate more extreme effects of attention on the speed of evidence accumulation. Given $\theta = 1$, the aDDM reduces to a standard DDM, meaning that attention does not amplify evidence accumulation. The aDDM was used to simulate 100 responses to each choice problem and for each parameter setting in order to reduce nonsystematic variability. This yielded a total of 5 (settings of θ) × 11 (settings of attentional bias) × 500 (choice problems) × 100 (responses per problem) = 1,100,000 choices. For all simulations the nonattentional parameters of the aDDM were set to d = 0.0002 and $\sigma = 0.02$ based on the estimates obtained by Krajbich et al. (2010). The upper and lower decision boundary were set at a distance of ±1 from the starting point. For each simulated choice process several outcomes were recorded: the chosen option (A or B) and its value ($V_{chosen} = V_A$ or $V_{chosen} = V_B$), response time, accuracy (binary variable set to 1 if the option with the higher value was chosen and 0 if the option with the lower value was chosen), and the reward rate $RR = V_{chosen}/RT$. All simulated data and code to implement the simulations are available on OSF (Zilker, 2022, February 21).

3. Results of the simulation study

How did increasingly extreme attentional biases in the aDDM affect accuracy, response time, and reward rate—and how did this depend on other conditions assumed in the simulation?

3.1. Advantage of attentional biases depending on options' values

Fig. 1 shows the results for runs of the simulation in which attention was biased towards the higher-valued option ($Patt_{highV} \ge 0.5$). Overall, more severe attentional biases to the higher-valued option were linked to an increase in accuracy and a decrease in response time. Together, the increasing tendency to choose the higher-valued option and the reduction in response time culminated in an increased reward rate under more extreme attentional biases to the higher-valued option. That is, if attention was systematically biased towards the higher-valued option, stronger attentional biases were beneficial in terms of both accuracy and reward rate. This held for different levels of choice difficulty (value ratios) and for most values of θ —except for θ =1, where attention does not amplify evidence accumulation, such that the aDDM reduces to a standard DDM. The effects tended to be stronger given lower values of θ (i.e., a stronger attentional amplification of evidence accumulation in the aDDM) and, to some extent, given a higher value ratio (i.e., lower choice difficulty).

Fig. 2 shows the results for runs of the simulation in which attention was biased towards the lower-valued option ($Patt_{highV} \le 0.5$). Before inspecting these results in detail, briefly consider how such attentional biases to the lower-valued option can, in principle, affect the accumulation process in the aDDM: In the absence of attentional biases, RDV(t) on average drifts towards the higher-valued option (as evident from Eqs. 1–3). Attentional biases to the lower-valued option, or even in a switch in the direction (sign) of the drift, such that RDV(t) on average drifts

⁴ In this procedure, the minimum duration of fixating on an option is 1 ms (the duration of a single time step in the aDDM). Longer fixations emerge if the same option is sampled repeatedly in subsequent draws from the Bernoulli distribution. Therefore, this simulation procedure may result in a higher number of shorter fixations per trial than one would typically see in empirical data. However, additional simulations in Appendix A demonstrate that this does not systematically affect the results.



Fig. 1. Effects of increasingly severe attentional biases on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM) assuming that attention is biased to the higher-valued option in the choice problem. Results are displayed separately for different settings of the aDDM parameter θ and for different levels of choice difficulty (ratios between the options' values). Lower values of θ indicate a stronger attentional amplification of evidence accumulation.

towards the lower-valued option. If the drift is decelerated, the higher-valued option will still be chosen in a majority of cases (i.e., accuracy decreases but remains \geq 50%)—albeit less consistently. However, because the average drift towards the higher-valued option is now slower, response times will slow down, on average. If the drift switches direction, the lower-valued option will be chosen in a majority of cases instead (i.e., accuracy drops below 50%), because *RDV*(*t*) on average drifts towards the lower-valued option. With stronger attentional biases, this drift—and therefore also response times—speed up, on average.

With these patterns in mind, consider the simulation results. When attention amplified evidence accumulation (i.e., given $\theta < 1$), more extreme attentional biases to the lower-valued option (higher *att. bias* in Fig. 2) were consistently linked to a decrease in accuracy. This effect was stronger, and accuracy dropped below 50% faster, given a lower value of θ . This reflects that attentional biases counteracted the baseline drift towards the higher-valued option more strongly when θ was lower.

Next, consider the simulated response times (Fig. 2B). As long as accuracy remained \geq 50%, stronger attentional biases to the lower-valued option tended to be linked to an increase in response times (except if $\theta = 1$). This reflects that the attentional biases slowed the drift towards the higher-valued option, but had not yet switched its direction. Once accuracy dropped below 50%, stronger attentional biases to the lower-valued option were linked to a decrease in response time. This reflects that the direction of the average drift had switched, such that evidence on average accumulated in favor of the lower-valued option. This drift to the lower-valued option—and thus also the response time—became faster with stronger attentional biases. The effects of attentional biases to the lower-valued option on response times also depended on θ . Under lower values of θ , more extreme attentional biases were more consistently linked to a decrease in response time. Moreover, the diffects of attentional biases on the direction of the direction of the direction of the direction of the direction attentional biases were less consistently linked to a decrease in response time. Moreover, the effects of attentional biases on response time depended on choice difficulty (value ratio): When the value ratio was small, response times started to decrease given only relatively weak attentional biases. When the value ratio was larger, response times did not start to decrease until attentional biases were relatively severe. This reflects that the baseline drift towards the higher-valued option was stronger (and thus more difficult to counteract) the more distinct the options' values were.

The results regarding the reward rate mirrored the results regarding response times. Under low values of θ , the reward rate increased with more severe attentional biases. That is, in terms of reward rate, the decrease in time cost under stronger attentional biases compensated for the accompanying decrease in the tendency to choose the higher-valued



Fig. 2. Effects of increasingly severe attentional biases on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM) assuming that attention is biased to the lower-valued option in the choice problem. Results are displayed separately for different settings of the aDDM parameter θ and for different levels of choice difficulty (ratios between the options' values). Lower values of θ indicate a stronger attentional amplification of evidence accumulation.

option. This is consistent with the rationale implied by the marginal value theorem: In terms of reward rate, obtaining a less valuable option quickly can be preferable to investing more time to obtain a (potentially only slightly) more valuable option. Under higher values of θ (e.g., $\theta = 0.5$), the relationship between more severe attentional biases towards the lower-valued option and reward rate depended more heavily on choice difficulty: If the value ratio was low (and difficulty was high), stronger attentional biases to the lower-valued option were linked to an increase in the reward rate. When the value ratio was high (and difficulty was low), however, stronger attentional biases were linked to a decrease in reward rate, since they entailed both a decrease in the tendency to choose the higher-valued option and an increase in accuracy—can be linked to an increase in reward rate. This was more likely the more attention amplified evidence accumulation—that is, given lower values of the parameter θ —and given higher choice difficulty. Given higher values of θ and lower choice difficulty, attentional biases to the lower-valued option were instead linked to a decrease in the difficulty of the more attention amplified evidence accumulation—that is, given lower valued option were instead linked to a decrease in the advection and rate.

3.2. Advantage of attentional biases independent of direction

The previous section treated scenarios in which attention was biased to the higher-valued or lower-valued option separately. Since people typically do not know the values of options in a choice problem ahead of time, deliberately attending predominantly to either the higher- or lower-valued option is arguably not a psychologically plausible strategy. More realistically, people may strategically decide to display an attentional bias to any option, independent of its a priori unknown value. What are the normative consequences of displaying more severe attentional biases when their direction is independent of value? To address this question, Fig. 3 displays the results of the simulations aggregated across the different directions of attentional biases.

First, consider the results regarding accuracy (Fig. 3A). Given $\theta < 1$, more severe attentional biases were negatively related to accuracy. This is due to the fact that when the direction of attentional biases is random with respect to the options' values, an increased tendency to choose the predominantly attended option leads to increasingly random (and therefore less accurate) choice behavior (see Thomas et al., 2019). Moreover, more severe attentional biases were linked to faster response times, and this effect was more pronounced for lower values of θ and higher choice difficulty. Finally, more severe attentional biases (independent of value) also tended to be linked to an increase in reward rate



Fig. 3. Effects of increasingly severe attentional biases on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM) assuming that the direction of attentional biases does not depend on the options' values. Results are displayed separately for different settings of the aDDM parameter θ and for different levels of choice difficulty (ratios between the options' values). Lower values of θ indicate a stronger attentional amplification of evidence accumulation.

across most parameter settings (except $\theta = 1$); this effect was more pronounced for lower values of θ .

These analyses underscore that relying on biased information search—even if the decision maker is not initially aware of the value of the option towards which they predominantly attend—can help a decision maker be more efficient and achieve a higher reward rate.

3.3 Interplay between attentional biases and boundary separation

The previous analyses demonstrated that decision makers may achieve a higher reward rate by relying on more biased information search, which reduces the time cost of choice sufficiently to compensate for the reduction in accuracy. Another strategy for arriving at a choice faster is to consider less information overall. In the aDDM, this strategy could be implemented by assuming narrower choice boundaries. Given narrower choice boundaries, the model overall requires less evidence before making a choice, resulting in faster response times. Do the effects of attentional biases on accuracy, response times, and reward rates depend on differences in the boundary separation—and if so, how?

To address this question, the previous simulations were repeated while assuming a reduced boundary separation, such that the upper and lower decision boundaries were set at a distance of ± 0.25 from the starting point (instead of ± 1).⁵ Note that, as in the previous simulations, the parameters *d* and σ were fixed to specific values. Concurrently varying these parameters as well as the boundary separation may lead to identifiability issues. Fig. 4 displays the results, again considering the direction of attentional biases independent of the options' values. Relying on such narrow boundaries led to a considerable overall reduction in accuracy compared to the previous simulations, especially when θ was low. This reflects that the model barely samples any information on the options before making a choice. Moreover, the response times were considerably shorter: Mean response times ranged between 150 ms and 175 ms, compared to 1000 ms and 2500 ms in the previous simulations. Note that these response times were still modulated by attention; more severe attentional biases led to faster response times, especially when θ was low. Finally, due to the extreme reduction in response times, the reward rates were, overall, considerably higher than those in the previous simulations. Mean reward rates ranged between

⁵ Appendix A reports additional simulations with two additional settings of the boundary separation, which yielded qualitatively similar results.



Fig. 4. Effects of increasingly severe attentional biases on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM) assuming that the direction of attentional biases does not depend on the options' values. Results are displayed separately for different settings of the aDDM parameter θ and for different levels of choice difficulty (ratios between the options' values). Lower values of θ indicate a stronger attentional amplification of evidence accumulation. Results are based on simulations with a reduced boundary separation of ± 0.25 .

0.04 and 0.06, compared to 0.003 and 0.007 in previous simulations. Notably, similar to the response times, the reward rates were still modulated by attention: They increased with stronger attentional biases, in particular given lower values of θ . Overall, the highest reward rates in these simulations were obtained when the low boundary was combined with extreme attentional biases and very low values of θ . This illustrates that different frugal information search strategies, which can be formalized in distinct constructs in the aDDM—attentional biases and a lower overall amount of evidence gathered—concurrently shape how accurate, fast, and efficient a decision is. While the importance of setting appropriate boundaries in order to achieve a high reward rate has been discussed extensively in the literature (e.g., Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006; Bogacz, Hu, Holmes, & Cohen, 2010; Evans & Brown, 2017; Malhotra, Leslie, Ludwig, & Bogacz, 2018; Simen et al., 2009; Starns & Ratcliff, 2010) the impact of attentional biases has so far gone largely unnoticed.

Decision makers can employ an even more radical strategy than relying on narrow boundaries: They can forego information search entirely and simply guess. Appendix A demonstrates that whether guessing is more advantageous than gathering evidence before making a choice depends on choice difficulty (see also Oud et al., 2016).⁶ For instance, given the choice problems and parameter settings assumed in the simulations above, the reward rate expected under guessing is consistently higher than the reward rates achieved by the aDDM. However, if one relies on choice problems with more distinct values, the expected benefit of choosing nonrandomly increases and thus justifies a small amount of information search. That is, given sufficiently distinct options and appropriately narrow choice boundaries, the reward rate achieved by the aDDM can exceed the reward rate expected under guessing. Importantly, the analyses in Appendix A also demonstrate that in such a case, the effect of stronger attentional biases on the reward rate reverses: Given a boundary separation that leads to a higher reward rate than the one expected under guessing, stronger attentional biases carve out an important

⁶ This also depends on whether the choice problems are heterogeneous in terms of difficulty (see Malhotra et al., 2018; Moran, 2015, and the Discussion section).



5. Distribution of attention allocation in each data set. Vertical black lines mark the mean proportion of time spent attending to the higher-valued option, mean(prop.dwellhighv), in each data set

Fig.

constraint for stronger attentional biases to yield a benefit in terms of reward rate—namely, that the decision maker relies on an excessively wide boundary for the choice environment they operate in. Notably, several prior investigations have demonstrated that people notoriously rely on excessively wide boundaries (e.g., Bogacz et al., 2010; Evans & Brown, 2017; Oud et al., 2016; Starns & Ratcliff, 2010, 2012)—indicating that this condition for attentional biases to yield a benefit in terms of reward rate may often be met empirically.

4. Empirical analyses

The previous simulations established that stronger attentional biases—whether dependent on or independent of value—can be beneficial in terms of reward rate. They also demonstrated how this pattern depends on and interacts with differences in aDDM parameters and the structure of the choice environment. Nevertheless, the results obtained so far are purely theoretical. The parameter settings and choice problems assumed in the simulations are selective and represent only a thin slice of the vast range of conceivable scenarios. It is therefore unclear whether the obtained results generalize to empirical settings—that is, to investigations using different choice problems, and to participants who may be equipped with different parameter values than those assumed in the simulations. Do human decision makers who allocate their attention in a more biased manner tend to achieve a higher reward rate? How well does the empirical relationship between attention, accuracy, response time, and reward rate conform to the predictions of the aDDM? To address these questions, data from four previously published eye-tracking studies on preferential choice (Fiedler & Glöckner, 2012; Krajbich et al., 2010; Smith & Krajbich, 2018; Stewart et al., 2016) were reanalyzed.

4.1 Description of data sets

Overall, the analyses cover four datasets from studies conducted in different laboratories. All studies posed two-alternative forced-choice tasks from different choice domains (riskless choice between food items, risky choice with monetary outcomes, risky choice with nonmonetary outcomes). In each task participants repeatedly chose between two items with associated values while their eye movements were recorded. A brief description of each choice task is provided below; further details can be found in the original publications.

In the study by Krajbich et al. (2010), hungry participants first provided subjective ratings of several food items. In each trial of the subsequent choice task they made a binary choice between two of the positively rated food items. The ratings of the items included in the choice task ranged between 0 and 10. Dividing the rating of the chosen option by the response time yields the reward rate achieved by the response on each trial. The data were retrieved from GitHub (Molter & Thomas, 2019, January 25).

The experiment by Smith and Krajbich (2018) included choice tasks from several domains. In each trial of the food choice task participants chose between two food items that they had previously rated on a scale of 1 to 10 (analogous to the task used by Krajbich et al., 2010). Dividing the rating (i.e., value) of the option chosen in each trial by the corresponding response time yields the reward rate for each response. In each trial of the risky food choice task participants chose between two risky options. Each option offered the chance to win one of two previously rated food items; the probability to win each of the two food items within each option was 50%. The expected value of each risky option is given by the sum of the two food items' ratings divided by two. Dividing the expected value of the chosen option by the response time yields the reward rate achieved by the response in each trial. Finally, in each trial of the monetary risky choice task participants chose between two monetary amounts; the probability to win each option was 50%. The magnitude of each monetary amount



Fig. 6. Posterior attentional drift-diffusion model parameter estimates obtained for each data set. Colored dots represent subject-level estimates for individual participants. Black diamonds with errorbars represent the corresponding group-level posterior mean with 95% posterior interval.

was conveyed by the height of a bar displayed onscreen. The expected value of each risky option is given by the sum of the two monetary amounts divided by two. Dividing the expected value of the chosen option by the response time yields the reward rate achieved by the response on a trial. The expected values of options offered in the monetary risky choice task ranged from 69.678 to 250. Data from the choice tasks included in the experiment by Smith and Krajbich (2018) had to be analyzed separately, since the units of rewards obtained (and therefore the units of reward rate) differed between the tasks.⁷ The data were retrieved from the OSF (Smith, 2019, February 19).

In the study by Stewart et al. (2016), participants made choices between numerically described risky options with monetary outcomes. On each trial, both options offered the chance to win one nonzero monetary outcome with a corresponding probability, and nothing otherwise. The nonzero amount and the corresponding probability for each option were displayed onscreen. Dividing the expected value of the chosen option (obtained by multiplying the nonzero reward by its probability) by the response time yields the reward rate achieved by the response on a trial. The expected values of the options offered in the choice task ranged between and 40 and 450. The data were retrieved from GitHub (Stewart, 2020, November 30).

Fiedler and Glöckner (2012) conducted two experiments in which participants chose between numerically described risky options with monetary outcomes. Each trial consisted of two options, each of which offered two monetary outcomes with corresponding probabilities. The probabilities of the two outcomes within each option summed up to 1. Dividing the expected value of the chosen option by the response time yields the reward rate achieved by the response on each trial. Experiments 1 and 2 differed with regard to the specific choice problems used but otherwise relied on the same procedure (Fiedler & Glöckner, 2012). The expected values of the options offered in the choice task ranged between 0.95 and 9.90 in Experiment 1 and between 0.788 and 18.18 in Experiment 2. Because these different ranges of values imply different magnitudes of the reward rate (even given identical response times), data from each experiment are analyzed separately below. The data were retrieved from the OSF (Fiedler, 2017, October 5).

4.2 Preprocessing

The main analyses focused on data from all trials on which the options differed in their values. This is because accuracy cannot be determined in trials in which the options had exactly equal values (these trials were analyzed separately; see Appendix C). The current analyses covered 27,326 responses from 268 participants. Data from the studies and tasks were analyzed separately because they relied on different outcome units and ranges, and because response times are likely to differ between tasks in different domains as well as between tasks that pose varying demands. This results in different units and ranges for the reward rate, which make aggregation difficult, if not unwarranted. For each data set and each trial, the absolute dwell time on the option with the higher value, *abs. dwell*_{highV}, and on the option with the lower value, *abs. dwell*_{lowV}, were calculated. The absolute dwell time on any option is given by *abs. dwell*_{highV} + *abs. dwell*_{lowV} = *abs. dwell*_{highV}/*abs. dwell*_{highV}. The proportion of time fixating on the higher valued option (relative to time spent fixating on any option) is calculated as *prop. dwell*_{highV} = *abs. dwell*_{highV}/*abs. dwell*_{any}. The proportion of time fixating on the lower-valued option (relative to time spent fixating on any option) is calculated as *prop. dwell*_{highV} = *abs. dw*

abs. $dwell_{lowV}/abs. dwell_{any}$. The severity of attentional bias (regardless of which option attention is biased towards) is given by $att.bias = abs(prop. dwell_{highV} - 0.5)$. Like the measure employed in the simulations, att.bias ranges from 0 to 0.5, where 0 indicates no attentional bias (i.e., both options are attended to for equal amounts of time) and 0.5 indicates an extreme attentional bias (i.e., one of the options is attended to exclusively).

Fig. 5 displays the distribution of attention allocation in terms of *prop. dwell*_{highv} for each data set. Overall, moderate attentional biases tended to be more common than extreme ones. Nevertheless, it was also relatively common for people to exclusively attend to one of the options

⁷ The experiment by Smith and Krajbich (2018) also included a social choice task in which participants decided how to divide money between themselves and another participant. Data from this social choice task was not used for the subsequent analyses because the way in which participants evaluated the options differed between selfish and prosocial individuals (see Smith & Krajbich, 2018), making it difficult to consistently quantify the subjective value of the chosen division and hence the reward rate.

(i.e., *prop.* $dwell_{highV} = 0$ or *prop.* $dwell_{highV} = 1$)—particularly in the data from Smith and Krajbich (2018). All distributions are approximately centered on 0.5 (the mean of each distribution is marked by a vertical black line), indicating that on average, people tended to allocate their attention evenly between the two options, and that the direction of attentional biases did not seem to systematically depend on value. The corresponding distributions of attention allocation on the individual level (see Fig. C1 in Appendix C) are similarly symmetric.

4.3. Data-analytic approach

The data-analytic procedure combined statistical analyses and computational modeling to assess how stronger attentional biases were linked to accuracy, response times, and reward rate, and to test how well these empirically observed relationships conformed to the corresponding relationship predicted by the aDDM when assuming appropriate parameter values for each data set.

4.3.1. Computational modeling

Each data set was modeled using a hierarchical Bayesian implementation of the aDDM. The model was estimated using an approach proposed by Cavanagh et al. (2014), which is based on rewriting the aDDM's equations to describe the rate of change in the decision variable *RDV* (see Eq. 3), that is, the drift rate, δ , as

$$\delta = \beta_0 + \beta_1(gaze_B \cdot V_B - gaze_A \cdot V_A) + \beta_2(gaze_A \cdot V_B - gaze_B \cdot V_A).$$
⁽⁴⁾

In this formulation, the parameter θ can be obtained by taking the ratio β_2/β_1 , and β_1 equals the scaling constant *d* of the aDDM (for details see Cavanagh et al., 2014). β_0 is an intercept term that describes the drift rate before the amplifying impact of attention. V_A and V_B are the values of the two options (labeled A and B, where option B corresponds to the upper choice boundary), and $gaze_A$ and $gaze_B$ are the proportional dwell times for the two options. Relying on this approach makes it possible to express the likelihood function of the aDDM as a Wiener distribution rather than an accumulation process with discrete time steps, which greatly simplifies parameter estimation (for a related approach see Thomas et al., 2019). The starting point of the accumulation was fixed halfway between the decision boundaries. Besides θ and *d*, the boundary separation parameter, *a*, and the non-decision time, t_0 , were also freely estimated. In the hierarchical Bayesian implementation, participant-level parameters are informed by group-level distributions (Farrell & Lewandowsky, 2018; Kruschke, 2015). Separate group-level distributions were assumed for each empirical data set. The model was implemented using the Wiener module in JAGS (Wabersich & Vandekerchhove, 2014). It was estimated by running 30 chains of 25,000 samples each using the R2jags package (Su & Yajima, 2015). A total of 5,000 burn-in samples were discarded from the analyses. The potential scale reduction factor (Gelman & Rubin, 1992) was $\hat{R} \leq 1.01$ for all group-level parameters, indicating good convergence. A parameter recovery analysis, reported in detail in Appendix B, was conducted to demonstrate that differences in generative parameters can be reliably recovered based on this modeling approach. After fitting the model, the estimated subject-level parameters were used to generate posterior

Table 1

Coefficients and 95% posterior intervals for the Bayesian generalized linear mixed models for effects of the severity of attentional bias on accuracy, response time, and reward rate.

| Dependent variable | Study | Attentional bias to higher-valued option | Attentional bias to lower-valued option | All data |
|---|---|---|--|---|
| Accuracy | Krajbich et al. (2010) Smith and Krajbich (2018) Food risk Smith and Krajbich (2018) Monetary risk | 2.939 [1.668, 4.284] 2.874 [2.174, 3.597] 2.81 [2.196, 3.434] | -4.068 [-5.309, -2.859] -2.251 [-2.86, -1.642] -2.386 [-2.882, -1.897] | 0.172 [-0.585, 0.932] 0.499 [0.09, 0.914] -0.178 [-0.528, 0.157] |
| RT | Smith and Krajbich (2018) Food Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 Stewart et al. (2016) Krajbich et al. (2010) | 2.558 [1.982, 3.143] 4.887 [1.912, 8.051] 8.22 [5.474, 11.099] 5.6 [4.262, 6.931] | -1.258 [-1.746, -0.769] -5.7 [-8.854, -2.817] -6.048 [-8.232, -3.951] -7.813 [-9.505, -6.21] -0.103 [-9.368 0.165] | 0.695 [0.348, 1.049] -0.213 [-1.86, 1.458] -0.077 [-1.522, 1.347] -0.199 [-0.96, 0.555] -0.36 [-0.505] -0.214] |
| NI | Smith and Krajbich (2018) Food risk | -0.966 [-1.065, -0.865] | -0.947 [-1.074, -0.82] | -0.956 [-1.036, -0.878] |
| | Smith and Krajbich (2018) Monetary risk | -1.315 [-1.444, -1.189] | -1.065 [-1.169, -0.963] | -1.185 [-1.264, -1.106] |
| | Smith and Krajbich (2018) Food | -0.365 [-0.454, -0.277] | -0.398 [-0.503, -0.292] | -0.384 [-0.452, -0.317] |
| | Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 | -0.658 [-1.182, -0.132] -0.633 [-1.023, -0.24] | -0.45 [-0.94, 0.043] -0.559 [-1.018, -0.095] | -0.518 [-0.872, -0.15] -0.598 [-0.899, -0.296] |
| Reward rate | Stewart et al. (2016) Krajbich et al. (2010) Smith and Krajbich (2018) Food risk | -0.241 [-0.438, -0.042] 3.714 [2.8, 4.608] 5.166 [4.751, 5.579] | -1.379 [-1.57, -1.184] -0.314 [-1.429, 0.81] 3.043 [2.58, 3.507] | -0.9 [-1.037, -0.762] 2.914 [2.208, 3.616] 4.433 [4.125, 4.743] |
| | Smith and Krajbich (2018) Monetary risk | 163.084 [148.727, 177.3] | 105.245 [95.799, 114.651] | 134.12 [125.841, 142.188] |
| | Smith and Krajbich (2018) Food Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 Stewart et al. (2016) | 3.058 [2.46, 3.676] 0.866 [0.089, 1.661] 1.805 [0.987, 2.626] 108.972 [82.86, 135.245] | 1.581 [0.889, 2.277] 1.901 [1.161, 2.635] 1.598 [0.605, 2.592] 66.59 [42.123, 91.173] | 2.623 [2.164, 3.08] 1.337 [0.802, 1.871] 1.65 [1.016, 2.275] 90.764 [73.014, 108.825] |
| Reward rate (boundary controlled) | Krajbich et al. (2010) | 3.657 [2.785, 4.532] | -0.28 [-1.386, 0.842] | 2.892 [2.185, 3.589] |
| contoned) | Smith and Krajbich (2018) Food risk Smith and Krajbich (2018) Monetary risk | 5.118 [4.709, 5.525] 161.373 [147.416, 175.255] | 2.982 [2.517, 3.455] 103.915 [94.423, 113.334] | 4.407 [4.097, 4.719] 133.24 [125.156, 141.323] |
| | Smith and Krajbich (2018) Food Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 Stewart et al. (2016) | 3.01 [2.42, 3.605] 0.96 [0.226, 1.712] 1.812 [1.012, 2.621] 110.021 [83.878, 136.084] | 1.478 [0.769, 2.193] 1.792 [1.1, 2.491] 1.442 [0.477, 2.416] 62.526 [37.557, 87.277] | 2.592 [2.134, 3.046] 1.37 [0.864, 1.882] 1.627 [1.003, 2.247] 89.635 [71.989, 107.425] |

Note. Results shown separately for trials on which attention was biased towards the higher-valued option ($prop.dwell_{highv} > 0.5$), for trials on which attention was biased towards the lower-valued option ($prop.dwell_{highv} < 0.5$), and across all data. Boldface indicates credible effects.

Table 2

Coefficients and 95% posterior intervals for the Bayesian generalized linear mixed models for effects of the severity of attentional bias on accuracy, response time, and reward rate.

| Dependent variable | Study | Attentional bias to higher-valued option | Attentional bias to lower-valued option | All data |
|---|---|---|--|---|
| Accuracy | Krajbich et al. (2010) Smith and Krajbich (2018) Food risk Smith and Krajbich (2018) Monetary risk | 3.33 [1.731, 4.987] 3.186 [2.188, 4.223] 2.384 [1.456, 3.331] | -4.851 [-6.202, -3.487] -2.302 [-3.151, -1.477] -2.062 [-2.777, -1.353] | -0.282 [-1.183, 0.624] 0.387 [-0.188, 0.975] - 0.674 [-1.18, -0.171] |
| | Smith and Krajbich (2018) Food Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 Stewart et al. (2016) | 2.879 [1.916, 3.834] 5.657 [2.407, 9.135] 8.198 [5.428, 11.114] 5.869 [4.514, 7.216] | -2.166 [-2.961 , -1.376] -5.274 [-8.519 , -2.208] -6.048 [-8.216 , -3.927] -8.181 [-9.9 , -6.526] | 0.27 [-0.3, 0.846] 0.502 [-1.435, 2.477] -0.158 [-1.568, 1.304] 0.53 [-0.287, 1.359] |
| RT | Krajbich et al. (2010) Smith and Krajbich (2018) Food risk | -0.045 [-0.26, 0.169] -0.54 [-0.694, -0.39] | 0.041 [-0.254, 0.337] -0.605 [-0.78, -0.432] | -0.086 [-0.258, 0.083] -0.573 [-0.688, -0.458] |
| | Smith and Krajbich (2018) Monetary risk | -0.636 [-0.848, -0.426] | -0.529 [-0.692, -0.369] | -0.579 [-0.709, -0.453] |
| | Smith and Krajbich (2018) Food Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 | 0.013 [-0.143, 0.168] -0.621 [-1.213, -0.033] -0.574 [-0.974, -0.174] | 0.015 [-0.16, 0.187] 0.045 [-0.507, 0.61] - 0.553 [-1.003, -0.114] | 0.008 [-0.109, 0.124] -0.259 [-0.655, 0.141] -0.563 [-0.863, -0.262] |
| | Stewart et al. (2016) | -0.168 [-0.373, 0.037] | -1.261 [-1.486, -1.03] | -0.714 [-0.864, -0.565] |
| Reward rate | Krajbich et al. (2010) Smith and Krajbich (2018) Food risk Smith and Krajbich (2018) Monetary risk | 1.855 [0.776, 2.912] 2.862 [2.357, 3.389] 66.748 [49.114, 84.055] | -0.502 [-1.735, 0.75] 1.272 [0.722, 1.839] 41.298 [29.444, 53.04] | 1.533 [0.731, 2.335] 2.296 [1.911, 2.684] 51.602 [41.742, 61.435] |
| | Smith and Krajbich (2018) Food Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 Stewart et al. (2016) | 1.692 [0.684, 2.684] 0.644 [-0.206, 1.49] 1.854 [1.011, 2.684] 99.124 [72.244, 125.818] | 0.057 [-1.006, 1.157] 0.48 [-0.284, 1.25] 1.599 [0.59, 2.603] 47.171 [18.7, 75.893] | 1.119 [0.393, 1.874] 0.556 [-0.016, 1.127] 1.68 [1.041, 2.312] 81.714 [62.379, |
| Reward rate (boundary controlled) | Krajbich et al. (2010) | 1.845 [0.796, 2.891] | -0.453 [-1.694, 0.791] | 1.532 [0.72, 2.328] |
| controlledy | Smith and Krajbich (2018) Food risk Smith and Krajbich (2018) Monetary risk | 2.83 [2.306, 3.35] 66.19 [49.107, 83.405] | 1.213 [0.655, 1.771] 39.885 [28.215, 51.603] | 2.272 [1.878, 2.65] 50.893 [41.102, 60.67] |
| | Smith and Krajbich (2018) Food Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 Stewart et al. (2016) | 1.655 [0.636, 2.647] 0.745 [-0.071, 1.566] 1.867 [1.058, 2.684] 100.586 [74.57, 126.622] | -0.004 [-1.065, 1.068] 0.461 [-0.285, 1.192] 1.443 [0.497, 2.388] 43.708 [15.666, 71.689] | 1.103 [0.37, 1.828] 0.62 [0.057, 1.167] 1.665 [1.038, 2.293] 81.226 [62.178, 100.177] |

Note. Results of analyses in which trials with extreme attentional biases (*att. bias* =0.5) were excluded. Results shown separately for trials on which attention was biased towards the higher-valued option (*prop. dwell*_{highV} >0.5), trials on which attention was biased towards the lower-valued option (*prop. dwell*_{highV} <0.5), and across all data. Boldface indicates credible effects.

predictive choice behavior and response times for each data set. That is, a synthetic version of each empirical experiment was simulated using the participants' characteristic parameter estimates and the aDDM as the generative model. These posterior predictive data sets can be compared against the empirical data sets to assess how well the empirical relationship between attention, accuracy, response time, and reward rate conforms to the aDDM's predictions.

4.3.2. Statistical models

Bayesian generalized linear mixed models (GLMMs), estimated using the rstanarm package in R (Goodrich & Gabry, 2018), were used to statistically test whether more severe attentional biases were credibly linked to accuracy, response time, and reward rate-both in each empirical data set, and in each corresponding posterior predictive data set generated by the aDDM. All models included a random intercept for each participant. Random slopes were not included to avoid excess model complexity. The severity of attentional bias, att. bias, was included as a fixed effect. A logistic link function was used in models with accuracy (choice of the higher-valued option) as the outcome variable. The response times were log-transformed before being used as the outcome variable. In all models using the reward rate as the outcome variable the values of the reward rate were multiplied by 1,000 to adjust the scale of the resulting coefficients. The effect of att. bias on each outcome variable is considered credible if the 95% posterior interval of the coefficient excludes zero. Separate Bayesian GLMMs were computed for the different outcome variables (accuracy, response time, and reward rate), and for different subsets of the data. Specifically, each empirical data set, and each corresponding posterior predictive data set, was first split into trials in which attention was biased towards the higher-valued option (prop. dwell_{highV} >0.5) and trials in which attention was biased towards the lower-valued option (prop. dwell_{highv} <0.5). The GLMMs were fitted separately to subsets of data in which attention was biased to either the higher- or the lower-valued option. Since the symmetric distributions of attention (see Fig. 5 and Fig. C1) indicated that the direction of attentional biases did not systematically depend on value, the same GLMMs were subsequently estimated across all data from each experiment, that is, without treating trials in which attention was biased to the higher- or lower-valued option separately. Moreover, to test whether potential effects of attentional biases on accuracy, response times, and reward rate were mainly driven by extreme attentional biases, all analyses were repeated after excluding trials on which participants exclusively attended to one option (att. bias =0.5) from the respective subset of data.

These GLMM analyses yielded β -coefficients capturing the effect of



Fig. 7. Comparison of β-coefficients describing the effects of stronger attentional biases in the synthetic data generated using the attentional drift–diffusion model (aDDM; x-axis) and in the empirical data (y-axis). Error bars represent the 95% posterior intervals around the β-coefficients obtained when estimating the GLMMs. Panel A: Effects on accuracy. Panel B: Effects on response times. Panel C: Effects on reward rates.

attention on each outcome variable (accuracy, response time, reward rate) in each empirical data set, and in the corresponding synthetic data sets generated by the aDDM. To assess how well the empirical relationship between attention and the diverse outcome variables matched the aDDM's predictions, the β -coefficients obtained based on the empirical data were compared to the corresponding β -coefficients obtained based on the posterior predictive data.

5. Results of the empirical analyses

5.1. Parameter estimates

Fig. 6 displays the posterior mean estimates of the key parameters of the aDDM (attentional amplification θ , boundary separation *a*, scaling parameter *d*) for each data set. Consistent with the key assumption of the aDDM, the parameter estimates for θ were smaller than 1 in all but seven participants, indicating that attending more to an option amplified the accumulation of evidence towards this option. Across studies, θ tended to be lowest in the data from Stewart et al. (2016), indicating relatively strong attentional amplification, and highest in the data from Experiment 2 by Fiedler and Glöckner (2012), indicating relatively weak attentional amplification. The boundary separation *a* was comparably wide in the data sets from Fiedler and Glöckner (2012), likely reflecting that participants took a relatively long time to arrive at a choice in the comparably complex risky choice task, where each option consisted of several outcomes and probabilities. The estimates for the scaling parameter *d* were particularly low for the experiment by Stewart et al. (2016) and for the monetary risky choice task by Smith and Krajbich (2018). In these tasks the options had relatively high nominal values compared to the other experiments, which explains the stronger downscaling.

Taken together, the parameters differ substantially between participants and also depending on the features of the experiment. This underscores that it is important to compare the behavioral patterns in each empirical data set against the aDDM's predictions based on the appropriate parameter constellation. This is achieved by the current approach based on posterior predictives.

5.2. Attentional bias and accuracy

A first series of GLMM analyses tested how stronger attentional biases were empirically related to accuracy, and how well this relationship corresponds to the patterns predicted by the aDDM. Table 1 displays the coefficients and 95% posterior intervals for the effect of attentional bias on accuracy in different subsets of empirical data. In all seven empirical data sets, more severe attentional biases towards the higher-valued option were credibly and positively linked to a higher tendency to choose this option (i.e., accuracy), and more severe attentional biases towards the lower-valued option were credibly and negatively linked to the tendency to choose the higher-valued option. When viewed across all data (i.e., considering attentional biases independent of value), these opposing effects of attention on accuracy in the two subsets largely eliminated each other. That is, the effect of attentional biases on accuracy was credible in data from the food risk task and the food choice task by Smith and Krajbich (2018), but not in any of the other data sets. Table 2 displays the results for analogous analyses, when extreme attentional biases (*att. bias* =0.5) were excluded from the data. The results are robust to this exclusion, indicating that the effects of stronger attentional biases on accuracy are not predominantly driven by trials on which participants exclusively attend to one option.

How well do the empirical effects of attention on accuracy correspond to the aDDM's predictions? Fig. 7 A plots the β -coefficients reflecting the effect of attention on accuracy in each empirical data set against the corresponding effects obtained based on the posterior predictive data generated using the aDDM. The coefficients largely align with the diagonal in most data sets, indicating that the empirical observations relatively closely resemble the effect of attention on accuracy predicted by the aDDM. The correspondence tends to be highest in the full data and slightly weaker when only a subset of data (attention biased to higher-/lower-valued option) is considered.

5.3. Attentional bias and response time

How were stronger attentional biases empirically related to response times? Table 1 also displays the coefficients and 95% posterior intervals for the effect of attentional bias on response times in the different subsets of data. On trials in which participants predominantly attended to the highervalued option (*prop. dwell*_{highV} >0.5), increasingly severe attentional biases were credibly linked to a decrease in response time in all data sets. In trials in which participants predominantly attended to the lower-valued option (*prop. dwell*_{highV} <0.5), increasingly severe attentional biases were credibly linked to a decrease in response time in all data sets except those from Krajbich et al. (2010) and Experiment 1 from Fiedler and Glöckner (2012). When analyzed across all data (i.e., considering attentional biases independent of value), more severe attentional biases were credibly linked to a decrease in response time in all data sets. After excluding trials with extreme attentional biases, the effects of attention on response time were no longer credible in the data sets from Krajbich et al. (2010), the food choice task from Smith and Krajbich (2018), and Experiment 1 from Fiedler and Glöckner (2012) see Table 2), indicating that in these data sets, trials on which participants inspected only one option seem to drive the effect of attention on response times.

How well do the empirical effects of attention on response time correspond to the aDDM's predictions? Fig. 7 B plots the β -coefficients reflecting the effect of attention on response times in the empirical data against those obtained based on posterior predictive choice behavior generated by the aDDM. Most of the β -coefficients have the same signs in the empirical and the synthetic data. Nevertheless, the magnitude of the empirical effects tended, in some cases, to diverge considerably from those predicted by the aDDM. For instance, in the data sets from Smith and Krajbich (2018), the empirically observed effects of attentional biases on response time tended to be substantially less pronounced than predicted by the aDDM. Overall, these results indicate that although the reduction in response time under stronger attentional biases predicted by the aDDM was qualitatively present in the empirical data, there were notable quantitative deviations between the empirical and predicted effects.

5.4. Attentional bias and reward rate

How were stronger attentional biases empirically related to reward rates? Table 1 displays the coefficients and 95% posterior intervals for the effect of attentional bias on accuracy in the different subsets of data. Fig. 8 illustrates the association between the severity of attentional bias and reward rate in each subset of data. Stronger attentional biases to the higher-valued option were linked to a credible increase in reward rate in all data sets. Stronger attentional biases to the lower-valued option were linked to a credible increase in reward rate in all data sets except the one from Krajbich et al. (2010). In other words, in terms of reward rate, the increased tendency to choose the lower-valued option was typically compensated for by the decrease in time invested in the choices. Also, when analyzed across all trials, more severe attentional biases (independent of value) were credibly linked to an increase in reward rate in all data sets. These results are robust to excluding trials with extreme attentional biases (see Table 2), except in the food choice task from Smith and Krajbich (2018) and in Experiment 1 from Fiedler and Glöckner (2012). This indicates that in most data sets there was a beneficial effect of stronger attentional biases on the reward rate, and this effect was not typically driven by trials on which participants exclusively attended to one option.

How well do the empirical effects of attention on reward rate



Fig. 8. Relationship between severity of attentional bias and reward rate. Reward rates were rescaled consistent with the GLMMs. Panel A: Trials in which attention was biased towards the higher-valued option. Panel B: Trials in which attention was biased towards the lower-valued option. Panel C: All trials.

Table 3

Coefficients and 95% posterior intervals for the Bayesian generalized linear mixed models for effects of the boundary separation α on reward rate.

| | Study | Attentional bias to higher-valued option | Attentional bias to lower-valued option | All data |
|-----------------------------|--|--|---|--------------------------------|
| Including extreme biases | Krajbich et al. (2010) | -1.069 [-1.654, -0.499] | -0.794 [-1.263, -0.332] | -0.97 [-1.486, -0.484] |
| | Smith and Krajbich (2018) Food risk | -1.078 [-1.472, -0.694] | -1.053 [-1.378 , -0.73] | -1.06 [-1.402, -0.705] |
| | Smith and Krajbich (2018) Monetary risk | -33.752 [-41.892, -25.24] | -31.783 [-37.46, -25.883] | -33.7 [-40.645, -26.731] |
| | Smith and Krajbich (2018) Food | -1.717 [-2.512, -0.901] | -1.373 [-2.059, -0.693] | -1.551 [-2.299, -0.821] |
| | Fiedler and Glöckner (2012) Exp. 1 | -0.274 [-0.328, -0.221] | -0.261 [-0.314, -0.208] | -0.268 [-0.308, -0.228] |
| | Fiedler and Glöckner (2012) Exp. 2 | -0.213 [-0.257, -0.168] | -0.238 [-0.296, -0.182] | -0.224 [-0.269, -0.178] |
| | Stewart et al. (2016) | -14.568 [-22.07, -6.958] | -18.939 [-26.973, -10.931] | -15.757 [-22.9, -8.831] |
| Excluding extreme biases | Krajbich et al. (2010) | -0.989 [-1.517, -0.462] | -0.82 [-1.265, -0.358] | -0.923 [-1.413, -0.437] |
| | Smith and Krajbich (2018) Food risk | -1.02 [-1.356, -0.698] | -0.928 [-1.246, -0.608] | -0.976 [-1.284, -0.657] |
| | Smith and Krajbich (2018) Monetary risk | -29.677 [-36.043, -23.265] | -29.385 [-35.077, -23.696] | -29.512 [-35.437, -23.639] |
| | Smith and Krajbich (2018) Food | -1.767 [-2.506 , -1.017] | -1.377 [-2.042 , -0.698] | -1.586 [-2.282 , -0.898] |
| | Fiedler and Glöckner (2012) Exp. 1 | -0.272 [-0.326, -0.218] | -0.248 [-0.297, -0.198] | -0.26 [-0.299, -0.221] |
| | Fiedler and Glöckner (2012) Exp. 2 | -0.214 [-0.257, -0.169] | -0.238 [-0.297, -0.181] | -0.224 [-0.269, -0.178] |
| | Stewart et al. (2016) | -14.328 [-21.895, -6.697] | -17.883 [-25.759, -10.12] | -15.166 [-22.108, -8.201] |

Note. Results shown separately for trials on which attention was biased towards the higher-valued option ($prop. dwell_{highv} > 0.5$), for trials on which attention was biased towards the lower-valued option ($prop. dwell_{highv} < 0.5$), and across all data. Boldface indicates credible effects.

Table 4

Coefficients and 95% posterior intervals for the interactive effects of the severity of attentional bias and the parameter θ on reward rate.

| Study | Attentional bias to higher-valued option | Attentional bias to lower-valued option | All data |
|--|--|--|---|
| Krajbich et al. (2010) Smith and Krajbich (2018) Food risk Smith and Krajbich (2018) Monetary risk Smith and Krajbich (2018) Food Fiedler and Glöckner (2012) Exp. 1 Fiedler and Glöckner (2012) Exp. 2 | -7.364 [-10.58, -4.097] -5.574 [-7.316, -3.825] -92.948 [-130.474, -55.051] -8.323 [-11.162, -5.442] -0.455 [-1.634, 0.727] 2.51 [-3.06, 8.188] | 3.115 [-0.763, 6.974] -2.951 [-4.874, -1.078] -69.646 [-94.69, -44.328] -0.241 [-3.739, 3.287] -1.44 [-2.707, -0.186] 1.742 [-4.921, 8.576] (-0.91 [-0.99.090, 200, 742] | -5.062 [-7.6, -2.538] -4.604 [-5.903, -3.297] -69.847 [-91.831, -48.313] -6.146 [-8.363, -3.932] -0.933 [-1.773, -0.074] 2.151 [-2.138, 6.422] |
| biewait et al. (2010) | -515.51 [-576.200, -54.903] | -000.01 [-000.003, -220.740] | -516.766 [-525.242, -104.302] |

Note. Coefficients and posterior intervals obtained in Bayesian generalized linear mixed models separately for trials on which attention was biased towards the higher-valued option (*prop. dwell*_{*highV*} >0.5), for trials on which attention was biased towards the lower-valued option (*prop. dwell*_{*highV*} <0.5), and across all data. Boldface indicates credible effects.

correspond to the aDDM's predictions? Fig. 7C plots the β -coefficients reflecting the effect of attention on reward rate in the empirical data against those obtained based on posterior predictive data generated using the aDDM. The match between empirical and predicted coefficients is impressively high in the data from Krajbich et al. (2010). There are notable similarities between the empirical and predicted patterns in the other data sets as well: Almost all of the β -coefficients have the same signs in the empirical and the synthetic data. Moreover, the positive effect of attentional biases on reward rate tended to be strongest when considering only trials where attention was biased to the higher-valued option, and weakest when considering only trials where attention was biased to the lower-valued option. This holds in most empirical data sets and the corresponding data simulated using the aDDM. Regardless of these qualitative similarities, the magnitude of the empirical effects did not match the theoretically predicted ones perfectly in most data sets. This is likely a consequence of the imprecise match between the empirically observed and predicted response time patterns (see Fig. 7B). For instance, in data from Smith and Krajbich (2018), the empirically observed effects of attention on the reward rate overall tended to be weaker than predicted by the aDDM.

5.4.1. Interplay between attentional biases and boundary separation

The previous analyses demonstrated that stronger attentional biases tend to be linked to a higher reward rate in diverse sets of empirical data a pattern that aligns, at least qualitatively, with the aDDM's predictions for the same data sets. Remember from the initial simulations that, according to the aDDM, an increase in reward rate can emerge due to not only stronger attentional biases, but also a lower boundary separation. That is, if people who displayed more extreme attentional biases also relied on narrower boundaries, the observed increase in reward rate might be an artifact of such differences in the boundary separation. Therefore, it is important to test whether attentional biases still yield a benefit in terms of reward rate after controlling for individual differences in the boundary separation. To this end, the subject-level estimates of the boundary separation parameter a were included as additional fixed predictors in the GLMMs testing for an effect of att. bias on the reward rate in the empirical data. The resulting coefficients are displayed in the lowest section of Table 1, which shows that the effect of att. bias on the reward rate remained credible and was only very slightly reduced across all data sets when α was controlled for, compared to when it was not controlled for. When excluding trials with extreme attentional biases (see Table 2) the effects of att. bias on the reward rate were also robust to controlling for a. Appendix C reports additional analyses on the participant level, testing whether participants who on average tended to show stronger attentional biases also tended to achieve a higher average reward rate, and whether these effects were affected by controlling for the boundary separation. Table 3 displays the coefficients representing the effect of the boundary separation α on the reward rate. As expected, a higher boundary separation was linked to a lower reward rate throughout. This result was also robust to excluding trials with extreme attentional biases. The current analyses thus provide empirical evidence that distinct frugal search strategies-stronger attentional biases and lower choice boundaries-seemed to independently contribute to an increase in participants' reward rates.

Finally, remember that the simulations reported in Appendix A indicated that attentional biases may only be beneficial if people rely on

excessively wide boundaries in the first place—that is, if they could increase their reward rate by further lowering their choice boundaries. In this light, the finding that people seemed to benefit from relying on stronger attentional biases in the analyzed data sets indicates that, from a reward rate perspective, people overall tended to rely on overly wide boundaries.

5.4.2. Interplay between attentional biases and attentional amplification θ

The previous analyses indicated that stronger attentional biases can have a beneficial effect on the reward rate, even after controlling for the boundary separation parameter α . Does the beneficial effect of attentional biases on the reward rate depend on the second key parameter of the aDDM, the attentional amplification θ ? The initial simulations indicated that attentional biases can have beneficial effects on the reward rate, particularly in people with a lower value of θ (i.e., attention more strongly amplifies evidence accumulation; see Fig. 3). Is this also the case empirically?

Additional GLMM analyses were conducted to test for interactive effects between the severity of attentional bias and θ on the reward rate. In addition to the main effects of severity of attentional bias and of *a* these GLMMs also included a main effect for the subject-level estimates for θ , and the interaction between the severity of attentional bias and θ .

Table 4 displays the resulting estimates for the interaction effect between attentional biases and θ . In trials in which participants predominantly attended to the higher-valued option (*prop. dwell*_{highV} >0.5), the interaction between severity of attentional biase and θ was credible and negative in all data sets except those from Fiedler and Glöckner (2012). This indicates that participants with lower values of θ benefited more from implementing stronger attentional biases in terms of reward rate. In trials in which participants predominantly attended to the lower-valued option (*prop. dwell*_{highV} <0.5), the interaction was credible and negative in all data sets except for Krajbich et al. (2010), the food choice task by Smith and Krajbich (2018), and Experiment 2 by Fiedler and Glöckner (2012). When analyzed across all data—that is, when considering attentional biases independent of value—the effect was credible in all data sets except for Experiment 2 by Fiedler and Glöckner (2012).

How well do these empirical interactions between severity of attentional biases and θ conform to the aDDM's predictions? Fig. 9A compares the β -coefficients capturing the interactive effect between attention and θ in the empirical data against the corresponding interactive effects obtained based on posterior predictive behavior simulated using the aDDM. The coefficients closely align with the diagonal, indicating that the empirically observed interactions closely conformed to the aDDM's predictions.

Overall, these analyses indicate that there are individual differences in how beneficial attentional biases are in terms of reward rate, depending on a person's characteristic parameter θ . People with a lower θ —that is, for whom attention more strongly amplifies the accumulation of evidence—tended to achieve larger gains in reward rate when implementing attentional biases than did participants with a higher θ .

5.4.3. Interplay between attentional biases and choice difficulty

The next series of analyses tested whether the beneficial effects of attentional biases on reward rate depended on features of the choice environment—particularly, choice difficulty. Remember that the initial simulations indicated that stronger attentional biases may entail a stronger increase in reward rate in choice problems where the options' values are more similar (all else being equal)—at least when attention was biased to the lower-valued option. Were the empirical effects of attentional biases on reward rate modulated by choice difficulty?

Additional GLMM analyses were conducted to test for interactive effects between the severity of attentional bias and choice difficulty on the reward rate. In addition to the main effects of severity of attentional bias and of α , these GLMMs also included a main effect for choice difficulty (measured in terms of the ratio between the options' values,



Fig. 9. Comparison of β -coefficients. Error bars represent the 95% posterior intervals around the β -coefficients obtained when estimating the GLMMs. Panel A: Comparison of β -coefficients describing the interactive set of the Panel B: Comparison of β coefficients describing the interactive effects of stronger attentional biases and choice difficulty on reward rate in the synthetic data generated using the aDDM (x-axis) and in the empirical data (y-axis). and heta on reward rate in the synthetic data generated using the attentional drift-diffusion model (aDDM; x-axis) and in the empirical data (y-axis). effects of stronger attentional biases

Table 5

Coefficients and 95% posterior intervals for the interactive effects of the severity of attentional bias and choice difficulty on reward rate.

| Study | Attentional bias to higher-valued option | Attentional bias to lower-valued option | All data |
|---|--|---|----------------------------------|
| Krajbich et al. (2010) | -0.48 [-1.308, 0.346] | -0.1 [-1.259, 1.025] | -0.121 [-0.793, 0.534] |
| Smith and Krajbich (2018) Food risk | -1.492 [-2.164, -0.816] | -2.528 [-3.291, -1.767] | -1.833 [-2.348, -1.32] |
| Smith and Krajbich (2018) Monetary risk | -37.694 [-142.25, 70.099] | -239 295 [-314.003, -163.702] | -124 854 [-187,779, -62,029] |
| Smith and Krajbich (2018) Food | -0.426 [-0.937, 0.077] | -0.317 [-0.949, 0.318] | -0.296 [-0.679, 0.092] |
| Fiedler and Glöckner (2012) Exp. 1 | -0.508 [-1.782, 0.752] | -1.459 [-3.147, 0.254] | -0.793 [-1.778, 0.203] |
| Fiedler and Glöckner (2012) Exp. 2 | -1.421 [-2.726 , -0.072] | -2.874 [-5.118 , -0.62] | -1.748 [-2.919 , -0.594] |
| Stewart et al. (2016) | 36.233 [6.467, 66.042] | -108.581 [-135.427 , -81.903] | -38.54 [-58.775 , -17.939] |

Note. Coefficients and posterior intervals obtained in Bayesian generalized linear mixed models separately for trials on which attention was biased towards the higher-valued option (*prop. dwell*_{highV} >0.5), for trials on which attention was biased towards the lower-valued option (*prop. dwell*_{highV} <0.5), and across all data. Boldface indicates credible effects.

analogous to the initial simulations) and the interaction between the severity of attentional bias and difficulty. The data from Krajbich et al. (2010) included trials on which the lower-valued option had a value of zero, resulting in infinitely high value ratios, which cannot be included as predictors in the GLMM analyses. These trials were therefore excluded from the current GLMMs.

Table 5 displays the resulting estimates for the interaction effect between attentional biases and choice difficulty. In trials in which participants predominantly attended to the higher-valued option (*prop. dwell*_{highV} >0.5), the interaction between value ratio and severity of attentional bias was credible in three out of seven data sets. In trials in which participants predominantly attended to the lower-valued option (*prop. dwell*_{highV} <0.5), the interaction was credible and negative in four of the seven data sets. That is, more severe attentional biases to the lower-valued option had stronger positive effects on reward rate when the value ratio between the options was lower. This is consistent with the notion that when the options' values differ less, there is less to gain from investing time in identifying the higher-valued option. A similar picture emerged in the analysis across all trials.

How well did these empirical observations conform to the aDDM's predictions? Fig. 9B compares the β -coefficients quantifying the interaction between attentional biases and choice difficulty obtained based on the empirical data to the corresponding coefficients obtained based on the posterior predictive data simulated using the aDDM. These β -coefficients aligned closely with the diagonal, indicating that the empirically observed interactions conformed well to the aDDM's predictions. Specifically, in data sets where the aDDM predicted a notable interaction effect, this effect tended to exist empirically. For empirical data sets in which no credible interaction effect was found, the aDDM typically did not predict one. That is, while the size and existence of interaction effects (Table 5) were relatively heterogeneous across the different data sets, the comparison to the aDDM revealed that such heterogeneity was actually predicted when assuming the parameter estimates most appropriate for each data set.

Overall, these analyses provide empirical evidence that whether and how strongly attentional biases are related to an increase in reward rate can depend on features of the choice environment, particularly on choice difficulty. The comparison between different studies also indicates that these interactive effects seem to depend on the overall parameter constellation of the aDDM.

6. Discussion

The current analyses investigate the implications of attentional biases in terms of two normative benchmarks: accuracy and reward rate. Whereas accuracy exclusively focuses on whether the higher-valued option was chosen, the reward rate makes it possible to assess the efficiency of strategies by taking into account their time cost. The aDDM was used as a theoretical framework to demonstrate that behavioral consequences of attentional biases that appear irrational when viewed solely in terms of accuracy maximization can be surprisingly beneficial when the time cost of making a decision is taken into account.

Conversely, choices based on unbiased attention can achieve a relatively high level of accuracy while often performing comparably poorly in terms of reward rate. While the beneficial effect of attentional biases on reward rate may be particularly strong and consistent if attention is systematically captured by and biased towards higher-valued items (Anderson, Laurent, & Yantis, 2011; Cavanagh, Malalasekera, Miranda, Hunt, & Kennerley, 2019; Gluth, Spektor, & Rieskamp, 2018; Le Pelley, Mitchell, Beesley, George, & Wills, 2016), the current analyses indicate that it can also emerge if attention is biased towards lower-valued options, and if the direction of attentional biases is independent of value. Attentional biases remained beneficial in terms of reward rate even after controlling for differences in the overall amount of information gathered (i.e., the boundary separation). Moreover, how beneficial attentional biases are in terms of reward rate depends both on features of the choice environment, such as choice difficulty, and of the decision maker, such as individual differences in the aDDM-parameter θ . These regularities are predicted theoretically by the aDDM; they also emerge empirically across various domains of both riskless and risky choice, and in data obtained by different groups of researchers. These results suggest that instead of considering all the available options evenly, decision makers may achieve a higher reward rate when they predominantly focus their attention on one option and deliberately ignore information on the other.

6.1. A novel sequential sampling perspective on reward rate maximization

Prior research on reward rate optimization in the context of the sequential sampling framework has often revolved around the question of optimal boundary setting—that is, how much evidence people require (or should require) before they make a choice (e.g., Bogacz et al., 2010; Evans & Brown, 2017; Malhotra et al., 2018; Simen et al., 2009; Starns & Ratcliff, 2010). This is because it is often assumed that the decision maker can strategically control how much evidence they consider sufficient to make a choice (i.e., the boundary separation), whereas the drift rate is mainly assumed to be a function of stimulus discriminability (Bogacz et al., 2010; Simen et al., 2009). In contrast, from the theoretical perspective offered by the aDDM, participants control the drift rate itself to some extent (assuming that they control their attention allocation). The model therefore offers a complementary perspective on how reward rate might be modulated: by searching for information in a biased manner, thus speeding up the decision process. This implies that, even given the same boundary separation, decision makers who rely on different attentional policies can achieve different reward rates. Consistent with this implication, the current simulations and empirical analyses provide evidence that differences in boundaries and attentional biases have dissociable effects on the achieved reward rates.

Although the aDDM warrants more strategic flexibility than, for instance, the classical drift diffusion model (DDM; Ratcliff, 1978), the normative inferences drawn are still conditioned on the constraints imposed by this formal framework. For instance, in choice environments like those investigated here, where the choice problems vary in difficulty, the optimal strategy is often to implement a time-varying decision

boundary (Drugowitsch, Moreno-Bote, Churchland, Shadlen, & Pouget, 2012; Malhotra et al., 2018; Moran, 2015). However, neither the classical DDM nor the aDDM assume a time-varying decision boundary. It may therefore not even be possible to implement a strictly optimal strategy that maximizes reward rate in the current theoretical framework. The current research should thus not be interpreted as claiming that attentional biases constitute such a strictly optimal strategy (for investigations taking a stronger stance on the optimality of attentional policies see, e.g., Callaway, Rangel, & Griffiths, 2021; Jang, Sharma, & Drugowitsch, 2021; Najemnik & Geisler, 2005). Instead, the current work aims to identify how performance differs between the attentional policies people actually seem to rely on—even if this may not include a strictly optimal strategy.

6.2. When can decision markers benefit from attentional biases?

Based on the current analyses neither the aDDM as a formal framework nor attentional biases can be considered strictly optimal in terms of reward rate. Nevertheless, there are systematic performance differences in terms of reward rate between the attentional policies people rely on, and whether attentional biases yield a benefit depends systematically on features of the decision maker and of the choice environment. This raises the question of which decision makers may benefit most from implementing an attentional bias, and in which situations.

6.2.1. Characteristics of the decision maker

The analyses presented here demonstrate that characteristics of the decision maker shape whether and how much they can benefit from implementing attentional biases. Specifically, people with a lower θ can achieve a more substantial reduction in response time, and thus a stronger increase in reward rate, when they rely on attentional biases compared to people with a higher θ . Notably, participants whose choice behavior is driven by attention allocation to a higher degree (i.e., who have a lower θ) have also been found to perform worse at choosing the best option in the choice set (see Thomas et al., 2019). These findings complement each other, showing that while a low θ may be detrimental in terms of accuracy, it can be beneficial in terms of reward rate. Whereas decision makers with a low θ may rely on attentional biases to modulate their reward rate, decision makers with a higher θ (or even $\theta = 1$) may have to rely on adjusting their boundary separation alone.

Moreover, the simulations indicate that attentional biases may predominantly benefit people who tend to rely on boundaries that are too cautious for their environment (see Appendix A). For these decision makers, attentional biases can be an effective strategy to counteract their tendency to gather excessive amounts of evidence. In contrast, attentional biases may be detrimental to decision makers who already rely on choice boundaries that are sufficiently narrow to yield a reward rate that exceeds the reward rate expected under guessing. Given the boundaries people relied on in the analyzed studies, attentional biases seemed to be of benefit. This indicates that, from a reward rate perspective, participants in these studies tended to rely on overly cautious boundaries. This is consistent with previous research demonstrating that people often prioritize accuracy over speed in a suboptimal manner (e.g., Bogacz et al., 2010; Evans & Brown, 2017; Oud et al., 2016; Starns & Ratcliff, 2010, 2012).

6.2.2. Environmental characteristics

Besides characteristics of the decision maker, characteristics of the choice environment shape whether and how much people can benefit from implementing attentional biases. Both the simulations and the empirical analyses highlight that partly or fully ignoring information on one of the options can be particularly beneficial when the options' values are more similar (see also Oud et al., 2016). Conversely, attentional biases may be less beneficial in easy choice problems, where the options have distinctive values. Notably, choice problems in preferential choice experiments are often designed to be nontrivial—that is, relatively difficult. Such experiments may therefore unintentionally create an incentive for people who wish to be efficient (in terms of reward rate) to rely on strategies such as biased information search.

Besides choice difficulty, the effects of attentional biases on reward rate also depend on the domain of outcomes offered by the choice environment. Additional simulations presented in Appendix A demonstrate that in an environment offering aversive options (losses instead of gains), stronger attentional biases are still linked to a decrease in accuracy—but in contrast to the positive environment, they are also linked to a decrease in reward rate. This reflects that decision makers who choose faster due to attentional biases collect more negative outcomes in a shorter amount of time. Therefore, in aversive environments like the one constructed for these simulations decision makers may achieve a higher reward rate by relying on balanced rather than biased search.

Further features of choice environments not covered in the current analyses may modulate how beneficial attentional biases are. For instance, in lab experiments each choice is typically followed almost immediately by the next choice problem. However, time lags between choices may have to be taken into account when assessing the reward rate, especially when opportunities to choose between rewarding options are sparse (e.g., Haith, Reppert, & Shadmehr, 2012; Moran, 2015). In such situations it may be more sensible for decision makers to use the time available between rare opportunities to make choices to identify the highest-valued option. Likewise, mainly attending to information on one option in order to emphasize speed may be less beneficial in such scenarios.

6.3. Ecological validity of normative inferences

The current analyses assessed the normative consequences of people's behavior in the confined context of lab experiments. It is important to keep in mind that such normative inferences may change drastically when taking into account larger contexts, including people's real-life choice environments. For instance, take a participant who strictly maximizes accuracy in the experiment and seems to perform poorly in terms of reward rate. If the rewards they gain in the experiment (e.g., the reimbursement and potential bonus payments earned) are greater than the rewards they might have obtained outside the lab in the same amount of time, then their behavior—including the mere act of participating in the experiment—may actually be a highly efficient strategy in terms of reward rate. However, this only becomes evident when taking into account information on the participant's choice environments outside the lab, which is often not available to researchers. Alternatively, researchers could attempt to apply principles of representative design (Brunswik, 1956) when constructing choice problems and reward structures for experiments in order to warrant more ecologically valid normative inferences.

6.4. Alternative formal frameworks

The current work relied on the aDDM, a relatively simple yet empirically successful model, to investigate the relationship between attention allocation and reward rate. However, the aDDM is not the only model of the relationship between attention allocation, preference, and response time (for other approaches see, e.g., Callaway et al., 2021; Gluth, Kern, Kortmann, & Vitali, 2020; Shimojo et al., 2003; Song, Wang, Zhang, & Li, 2019). The key finding—the often positive relationship between severity of attentional bias and reward rate—arises as a consequence of two well established empirical observations: the associations between attentional biases and preference and between attentional biases and response time. Other models that account for these two findings therefore likely also predict—at least qualitatively—a similar association between attentional biases and reward rate.

Although many aspects of the reported findings might be accommodated in other formal frameworks, one controversial feature

(Mormann & Russo, 2021) of the aDDM has important consequences for interpreting the current results. The model posits that attention causally shapes preference and response time via a mechanism that can be summarized as *gaze amplifies value* (Smith & Krajbich, 2019). Several other models also assume a causal effect of attention on preference (e.g., Gluth et al., 2020; Shimojo et al., 2003), and some empirical investigations have demonstrated that manipulating attention can affect choice behavior (e.g., Armel et al., 2008; Newell & Le Pelley, 2018; Pärnamets et al., 2015; Shimojo et al., 2003). These findings indicate that there may indeed be a causal link between attention and preference, which—in the light of the current results—could be exploited strategically to enhance reward rate. That is, people might enhance their reward rate by deliberately searching for information in a biased manner before making a choice. Nevertheless, it is important to note that none of the currently analyzed experiments involved an explicit manipulation of attention allocation. Therefore, despite being guided by the aDDM's predictions, the presented analyses cannot rule out the possibility that the observed empirical link between more severe attentional biases and increased reward rate is coincidental rather than causal.

6.5. How might attentional biases emerge?

While the current analyses indicate that there is notable variability in the strength and direction of the attentional biases people display (see Fig. 5), it is not clear how attentional biases come about. Attention may be driven by stimulus properties and perceptual factors such as the size, salience, or position (Orquin & Loose, 2013), which may create an advantage for a particular option. However, experimental studies—especially those employing eye-tracking—typically make an effort to control for such perceptual factors. More goal-relevant factors may also play a role in attentional biases. For instance, people tend to search more information on options whose outcome distributions are more variable (Lejarraga, Hertwig, & Gonzalez, 2012; Pachur & Scheibehenne, 2012), potentially reflecting a desire to reduce uncertainty about the options' values (Callaway et al., 2021; Jang et al., 2021). This may lead to option-specific attentional biases when one option in a choice problem is more uncertain than others. Moreover, options have been shown to capture attention proportional to their value (Anderson et al., 2011; Gluth et al., 2018; Le Pelley et al., 2016), and this can lead to fixation biases emerging over the course of a trial towards the more attractive options in a choice set (Gluth et al., 2018). A model developed by Callaway et al. (2021) combines these two notions and posits that attention is predominantly directed to options whose value estimates are both high and uncertain. Another possibility is that attentional biases may be induced by the use of heuristics, which prescribe distinctive patterns in information search and often systematically ignore some pieces of information (cf. Gigerenzer et al., 1999; Orquin & Loose, 2013).

6.6. Attentional biases and reward rates in risky choice

Five of the empirical data sets analyzed here were based on risky choice tasks. Some aspects of the analyses and results that apply exclusively to decisions under risk warrant further discussion, including the question whether reward rates in risky choice tasks should be assessed based on expected values or expected utilities, and the question how attentional biases and reward rates relate to distortions in the treatment of probabilities.

6.6.1. Reward rates versus subjective utility rates

Because people are typically not risk neutral, one could argue that in risky choice tasks, one should assess how stronger attentional biases are linked to the subjective utility rate (the subjective utility of the chosen option divided by the response time), rather than the reward rate (the expected value of the chosen option divided by the response time). This approach was not employed here because none of the analyzed data sets provided subjective ratings of the risky options' subjective utilities, meaning that these utilities would have to be inferred from responses in the choice tasks themselves. If one assumes that attention allocation shapes choice behavior (as the aDDM does), and derives utility function parameters from observed choice behavior, then these parameters and the inferred subjective utilities are already a function of attention. Arguably, by attending more to one option (thereby becoming more likely to choose it) a participant could thus increase the subjective utility of that option. As a byproduct, their subjective utility rate would almost necessarily increase, even if the response time remained stable. Put differently, if the utility of an option increases the more it is attended to, it may become almost trivial to show that stronger attentional biases are linked to a higher utility rate. This problem is circumvented by relying on the reward rate, where the expected value in the numerator is genuinely independent from the attentional process. Alternatively, one might rely on an external task to measure risky options' subjective utility—for instance, by asking participants to rate or bid on them—in order to render the assessment of utilities independent of the attentional process in the choice task. However, this raises its own problems, given a host of evidence indicating that the utilities that people ascribe to options differ systematically between tasks with different response modes, such as bidding versus choice (e.g., Lichtenstein & Slovic, 1971).

6.6.2. Relationship to nonlinear probability weighting

The current results also have implications regarding the notion of nonlinear probability weighting. Zilker and Pachur (2021) recently demonstrated that more severe attentional biases in risky choice are linked to more nonlinear probability weighting. A nonlinear probability weighting function—a central construct in cumulative prospect theory (Tversky & Kahneman, 1992), one of the most prominent theories of risky choice—indicates that behavior systematically deviates from the supposedly rational standard of expected utility maximization (which requires linear weighting). If stronger attentional biases are linked to more nonlinear probability weighting and to an increase in reward rate, then more nonlinear probability weighting may itself be associated with a gain in reward rate. This is particularly plausible since more nonlinear probability weighting also tends to be linked to lower response times (Zilker & Pachur, 2021). The demonstrated association between attentional biases and reward rate might, to some extent, rationalize choice patterns which give rise to nonlinearities in probability weighting. In other words, nonlinear probability weighting—conventionally considered a hallmark of irrational preference—might arise as a consequence of efficient strategies that enhance the decision maker's reward rate, and therefore have an adaptive benefit in situations where time is valuable.

7. Conclusion

Whether behavioral tendencies are considered rational or irrational

depends on the normative benchmark against which they are evaluated (e.g., Cohen, 1979). Some research traditions, such as behavioral economics, have even been characterized as having a "bias bias": an exaggerated tendency to attribute behavioral regularities that do not obey expected utility theory (e.g., Tversky & Kahneman, 1974, 1992; see Starmer, 2000, for an overview) to flaws of the human mind instead of questioning the normative framework (Gigerenzer, 2018). The current research identifies a case in which two normative benchmarks, accuracy and reward rate, can lead to opposite normative conclusions. The objective to maximize accuracy—choosing the option with the highest expected value or expected utility—is deeply ingrained in prominent economic theories. Maximizing accuracy is one of the most widely applied criteria for decision quality. Since stronger attentional biases tend to be associated with lower accuracy, it seems straightforward to conclude that such attentional biases are a sign of flawed decision making. However, the opposite conclusions can be drawn when considering the implications of attentional biases may help the decision maker strike a balance between obtained rewards and the time cost of decision making. Biased information search may be a feature of efficient decision making, rather than a bug that prohibits decision makers from maximizing accuracy.

Declarations of competing interest

None.

Availability of code and data

Data from all synthetic experiments and code to implement all analyses is available on the OSF http://doi.org/10.17605/OSF.IO/2GPVR (Zilker, 2022, February 21). Data for the empirical analyses is made available by the authors on the OSF (Smith, 2019, February 19, Fiedler, 2017, October 5) and on GitHub (Molter & Thomas, 2019, January 25, Stewart, 2020, November 30).

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Appendix A. Additional simulation results

A.1. Alternative visualization of simulation results

Fig. A1 provides an alternative visualization for the results of the simulations displayed in Fig. 1 and Fig. 2 in the main text. It displays the effects of differences in attention allocation on accuracy, response time, and reward rate on a continuous scale varying from strong biases to the lower-valued option over balanced allocation of attention to strong biases to the higher-valued option.



Fig. A1. Effects of differences in attention allocation, varying from strong biases to the lower-valued option over balanced allocation of attention to strong biases to the higher-valued option, on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM). Results are displayed

separately for different settings of the aDDM parameter θ and for different levels of choice difficulty (ratios between the options' values). Lower values on θ indicate a stronger attentional amplification of evidence accumulation.

A.2. Simulations in the loss domain

All choice problems in the simulations reported in the main text consisted of options with positive values. How are attentional biases related to accuracy, response time, and reward rates when choice problems offer negative values instead, that is, given aversive options? To investigate this question an additional simulation was conducted. All choice problems used for the simulations reported in the main text were mirrored into the negative domain by multiplying their outcomes by -1. Otherwise this new simulation followed the same procedure as the initial simulation reported in the main text. Fig. A2 displays the resulting effects of stronger attentional biases (independent of value) on accuracy, RT, and reward rate in the negative domain. Resembling the results obtained in the simulations in the positive domain, stronger attentional biases in the negative domain, stronger attentional biases were now associated with a decrease in response time. However, in contrast to the simulations in the positive domain, stronger attentional biases were now associated with a decrease in reward rate. This reflects that, by choosing faster, the decision maker obtains more aversive outcomes in a shorter amount of time. That is, these analyses indicate that, in order to increase their reward rate in a negative environment, decision makers should not implement attentional biases which speed up the decision process, but rather rely on more balanced attention during information search—at least given the parameter values and the reward structure investigated in the current simulations. Thereby, these analyses identify an important feature of the environment (domain of outcomes) which modulates whether attentional biases are beneficial or detrimental in terms of reward rate.



Fig. A2. Effects of increasingly severe attentional biases on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM) assuming that the direction of attentional biases does not depend on the options' values, and based on choice problems from the loss domain. Results are displayed separately for different settings of the aDDM parameter θ and for different levels of choice difficulty (ratios between the options' values). Lower values on θ indicate a stronger attentional amplification of evidence accumulation.

A.3. Alternative procedure to simulate the fixation process

The procedure used to simulate the fixation process in the simulations reported in the main text can give rise to some fixation patterns that may seem empirically implausible. Specifically, this procedure is likely to lead to a higher number of shorter fixations than one would typically see in empirical data because the minimum fixation duration is 1 ms. To test if this affects the resulting behavioral patterns, additional simulations were conducted using a procedure that avoids the emergence of a high number of extremely short fixations, thus giving rise to more empirically plausible fixation patterns. Again, a series of draws from a Bernoulli distribution was used to determine the fixation sequence. However, in contrast to the simulations process in the aDDM. The option sampled on this draw mas then assumed to be fixated for the following 50 ms, until a new sample was drawn. Therefore, each fixation had a minimum duration of 50 ms. If the same option was sampled repeatedly on *x* subsequent draws, the fixation duration was *x* · 50 ms. Based on the resulting fixation sequences, the aDDM was again used to simulate choice behavior and response times, based on the same procedure, choice problems, and parameter settings used for the simulations reported in the main text. The results are displayed in Fig. A3 and Fig. A4. Comparing these results to Fig. 1 and Fig. 2 in the main text demonstrates that the alternative assumptions regarding the fixation process have no systematic effects on the behavioral patterns in terms of accuracy, RTs, and reward rates.



Fig. A3. Effects of increasingly severe attentional biases on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM) assuming that attention is biased to the higher-valued option in the choice problem. Results are displayed separately for different settings of the aDDM parameter θ and for different levels of choice difficulty (ratios between the options' values). Lower values on θ indicate a stronger attentional amplification of evidence accumulation. Results are based on simulations assuming a lower number of longer fixations.



Fig. A4. Effects of increasingly severe attentional biases on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM) assuming that attention is biased to the lower-valued option in the choice problem. Results are displayed separately for different settings of the aDDM parameter θ and for different levels of choice difficulty (ratios between the options' values). Lower values on θ indicate a stronger attentional amplification of evidence accumulation. Results are based on simulations assuming a lower number of longer fixations.

A.4. Varying the boundary separation

Additional simulations were conducted to further explore how attentional biases and the boundary separation α concurrently modulate accuracy, response times, and reward rates. The boundary separation *a* was varied systematically within [0.25, 0.5, 0.75, 1.0]. Otherwise, the simulations were based on the same procedure as those reported in the main text. Fig. A5 displays the results of the simulations, such that the direction of attentional biases is independent of value. Given lower boundaries, behavior becomes both faster and also less accurate. This reflects that given lower boundaries, the model overall acquires less evidence before making a choice. Therefore, choice behavior depends less on the options' values and becomes increasingly random. Moreover, narrower boundaries entail an increase in reward rate (at least in the current reward environment). These regularities hold across the different settings of the boundary separation investigated here.



Fig. A5. Effects of increasingly severe attentional biases on accuracy, response time (RT), and reward rate, simulated in the attentional drift-diffusion model (aDDM). Results are displayed separately for different settings of the aDDM parameters θ and *a* and for different levels of choice difficulty (ratios between the options' values). Lower values on θ indicate a stronger attentional amplification of evidence accumulation.

A.5. Comparison to guessing

In some situations guessing—that is, not searching for any information before making a choice and choosing randomly—might lead to a higher reward rate than investing more time to gather evidence before making a choice. This section compares the expected reward rate under guessing to the reward rate achieved by the aDDM given different parameter settings and in different reward environments. The reward rate expected under guessing in a choice between two options A and B is given by the average value of the available options divided by the time it takes to implement a response without gathering any evidence, $mean(EV_A, EV_B)/RT_{guess}$. In the sequential sampling framework, the time it takes to implement a response without gathering any evidence, RT_{guess} , is commonly captured in the non-decision time τ . Notably, the aDDM in its original formulation by Krajbich et al. (2010) does not strictly assume a non-decision time, but there is a conceptually similar transitions time component, capturing transitions between fixations, which is added to the simulated reaction times. Depending on the available data, one might thus use the transition times between fixations, or potentially the latency to the first fixation, to derive values for the non-decision time. Because these latencies were not available for all currently analyzed empirical data sets, and because the estimated non-decision time to an approximate value of $\tau=200ms$. Note that this setting is relatively conservative (in the sense that it leads to relatively high reward rates under guessing), given that most non-decision times estimated in the empirical investigations were larger than 200 ms. This non-decision time τ is used to calculate the expected reward rate of guessing (assuming that $RT_{guess} = \tau$), and it is also added as a constant to the RTs predicted by the aDDM, to provide a fair comparison.



Fig. A6. Effects of increasingly severe attentional biases on reward rate, simulated in the attentional drift-diffusion model (aDDM, circles), compared to the expected reward rate of guessing (triangles). Results are displayed separately for different settings of the aDDM parameters θ and α and for different levels of choice difficulty (ratios between the options' values). Lower values on θ indicate α stronger attentional amplification of evidence accumulation.

Using the previously simulated data, we can now compare the expected reward rate of guessing to the reward rates achieved by the aDDM under different parameter settings of θ and α and under different levels of attentional biases. This comparison is displayed in Fig. A6. Circles indicate the reward rate achieved by the aDDM, and triangles of the same color indicate the expected reward rate under guessing for the same set of choice problems and parameters. As can be seen, none of the reward rates simulated by the aDDM using the current set of parameters exceed the corresponding reward rate expected under guessing. That is, although attentional biases entail an advantage in terms of reward rate over unbiased search, an even bigger advantage can be obtained by guessing in these simulated scenarios.



Fig. A7. Effects of increasingly severe attentional biases on reward rate, simulated in the attentional drift-diffusion model (aDDM, circles), compared to the expected reward rate of guessing (triangles). Results are displayed separately for different settings of the aDDM parameters θ and α . The ratio between the options' values (choice difficulty) was set to 10 for the displayed simulations. Lower values on θ indicate a stronger attentional amplification of evidence accumulation.

However, these analyses should not generate the misleading impression that guessing will *generally* achieve a higher reward rate than engaging in information search before a choice. Instead, whether this is the case depends on the structure of the choice environment. Specifically, if the options' values differ sufficiently, then the greater expected benefit of choosing non-randomly can justify some amount of time invested in information search.⁸ To demonstrate this, additional simulations were conducted, using a new set of choice problems with more distinct options. The problems were constructed in the same manner described in the main text, except that the value ratio between the options was set to 10. That is, the higher-valued option on each choice problem was 10 times as valuable as the lower-valued one. The parameter θ was varied within [0, 0.5, 1.0] and the boundaries were varied within distances of [±0.01, ±0.05, ±0.1, ±0.15, ±0.2] from the starting point. The aDDM was used to simulate 100 responses to each choice problem and for each parameter setting. Again, a non-decision time of $\tau=200ms$ was added to the RTs predicted by the aDDM, in order to ensure a fair comparison to the reward rates expected under guessing.

Fig. A7 compares the expected reward rate of guessing to the reward rates achieved by the aDDM in the new set of choice problems. Again, circles indicate the reward rate achieved by the aDDM, and triangles indicate the expected reward rate under guessing. Color codes for different settings of the choice boundary. In contrast to the previous simulations, the aDDM now exceeds the reward rate expected under guessing. This demonstrates that guessing is not generally preferable to engaging in information search in terms of reward rate. Instead, when the expected benefit of choosing non-randomly is sufficiently high (as in the current set of choice problems), a small amount of information search is justified. Note that for each level of θ , the highest reward rate is achieved when assuming a boundary of ± 0.1 and no attentional bias (*att. bias* =0). In comparison, the reward rate decreases both when assuming a narrower or a wider boundary separation (indicating that not enough or too much information is collected) and when assuming a more severe attentional bias. This is an interesting finding: When assuming boundary settings that yield higher reward rates than guessing in a given choice environment, stronger attentional biases no longer entail an increase, but a decrease in reward rate. Put differently, these simulations indicate that attentional biases seem to be beneficial when the decision maker relies on an excessively wide boundary for the environment they operate in, but not when their boundary is narrow enough to yield a reward rate that exceeds the one expected under guessing. That is, attentional biases may help decision makers achieve a higher reward rate in situations where they might also benefit from lowering their boundary separation. Thereby the current analyses delineate a constraint for attentional biases to benefit the decision maker in terms of reward rate.

⁸ To provide an intuition why this is the case, consider a choice problem in which both options' values are identical. The reward rate can only decrease by investing more time—meaning that the best thing the decision maker can do is to guess and respond immediately (see also Malhotra et al., 2018). Note that this simplified example presupposes, for the sake of the argument, that the decision maker knows ahead of time that the options' values are identical, and that all options in the choice set offer equal-valued options. Importantly, when the options become less similar in value, the expected benefit of choosing non-randomly increases, because choosing the higher-valued option results in a stronger increase in obtained reward. This justifies a greater investment of time in order to become more likely to choose the higher-valued option (Oud et al., 2016). This illustrates why the expected benefit of choosing non-randomly can be a function of the reward structure of the choice environment.

Appendix B. Parameter recovery

A parameter recovery analysis was conducted to demonstrate that the current implementation of the aDDM allows one to reliably estimate the model's parameters. To this end, the subject-level parameter estimates were used to generate posterior predictive choice behavior and response times for each empirical data set. For each subject, the empirically observed attentional patterns and choice problems were used as input to the model. The resulting data sets were structured exactly like the empirical ones. The aDDM was then fitted to this simulated data, relying on the same methods that were also used to model to the empirical data. Fig. B1 plots the generative subject-level posterior mean parameter values against the recovered posterior mean estimates thereof. As can be seen, the estimates tend to align quite closely with the diagonal, except for some rare outlier cases. This provides evidence that differences in generative parameters can be reliably recovered. Note that in the current implementation, the parameter σ determining the amount of noise, $\zeta(t)$, (see Eq. 1 and 2) is not freely estimated. The parameter recovery verifies that the scaling parameter d, the boundary separation α , and the attentional amplification parameter θ can be identified using this approach. To assess the recoverability quantitatively, the Pearson correlation between the generative and recovered subject-level parameter estimates was computed. This correlation was 0.9 for θ , 0.99 for α , and 0.97 for d, indicating a very high degree of recoverability. Since the subject-level estimates for the different studies cover various parameter values and combinations thereof, one can infer that differences in generative parameters can be reliably recovered across a wide range of possible parameter constellations.



Fig. B1. Results of the recovery of the subject-level aDDM parameters. A) Attentional amplification θ B) Boundary separation a C) Scaling parameter d.

Appendix C. Additional empirical analyses

C.1. Individual-level distributions of attention

Fig. C1 displays the distribution of attention allocation in terms of the proportion of time paid to the higher-valued option per trial in individual participants. Similar to the group-level distributions displayed in Fig. 5 in the main text the individual-level distributions tend to be centered approximately on 0.5 (the mean of each distribution is marked by a vertical black line), indicating that the direction of attentional biases did not seem to systematically depend on value. Whereas some participants tended to mostly show moderate biases, others were more inclined to attend to one option exclusively.



Fig. C1. Distribution of attention in individual subjects. Each subplot represents one participant. The prefix in the subplot heading identifies the experiment each individual participated in: $fi1_$ for Experiment 1 by Fiedler and Glöckner (2012). $fi2_$ for Experiment 2 by Fiedler and Glöckner (2012). $kr_$ for the experiment by Krajbich et al. (2010). $smi_rifo_$ for the risky food choice task by Smith and Krajbich (2018). $smi_rimo_$ for the monetary risky choice task by Smith and Krajbich (2018). smi_twofo_r for the food choice task by Smith and Krajbich (2018). st_r for the experiment by Stewart et al. (2016).

C.2. Interplay between attention and boundary separation on the participant level

Trial-level analyses reported in the main text demonstrated that the effect of the severity of attentional biases, att. bias, on the reward rate remained credible and was only very slightly reduced across all data sets when the boundary separation a was statistically controlled for, compared to when it was not controlled for. Additional analyses were conducted on the participant level. Did participants who on average showed stronger attentional biases also on average achieve a higher reward rate, and were such potential effects affected by controlling for the boundary separation? To test this, the average magnitude of att. bias and the average achieved reward rate was calculated for each participant. For each data set, a first GLM included the average reward rate as the outcome variable and the average attentional bias as a fixed predictor. A second GLM moreover included the subject-level estimates for the boundary separation a as a fixed predictor. Table C1 displays the resulting estimates for the effect of average attentional bias on the reward rate when a is not controlled for, and when a is controlled for. As can be seen, the effect of average attentional bias on the average reward rate was credible in the data by Fiedler and Glöckner (2012) and in the data from the risky food choice task and the monetary risky choice task by Smith and Krajbich (2018). This stands in contrast to the analyses on the trial-level reported in the main text, which identified credible effects of attentional biases on the reward rate in all data sets. This mismatch likely reflects a loss in statistical power in the participant-level analyses due to averaging across trials. Nevertheless, similar to the results obtained in the analyses on the trial-level, controlling for the boundary separation a did not eliminate the credible effect of attentional bias on the reward rate in the analyses on the participant-level in data Experiment 1 by Fiedler and Glöckner (2012) and the monetary risky choice task by Smith and Krajbich (2018). Only in the data from Experiment 2 by Fiedler and Glöckner (2012) and from the risky food choice task by Smith and Krajbich (2018) the effect of attention on reward rate was no longer credible after controlling for a.

Table C1

Coefficients and 95% posterior intervals for the effect of average severity of attentional bias on average reward rate on the participant level.

| Study | Boundary not controlled for | Boundary controlled for |
|---|---------------------------------|-------------------------------|
| Fiedler and Glöckner (2012) Exp. 1 | 7.997 [3.787, 12.348] | 2.659 [0.813, 4.519] |
| Fiedler and Glöckner (2012) Exp. 2 | 6.496 [0.175, 12.76] | 2.378 [-1.047, 5.941] |
| Krajbich et al. (2010) | -3.416 [-10.972, 3.803] | -3.663 [-9.828, 2.516] |
| Smith and Krajbich (2018) Food risk | 14.643 [9.861, 19.531] | 5.907 [-0.605, 12.363] |
| Smith and Krajbich (2018) Monetary risk | 380.59 [283.069, 474.957] | 174.606 [77.983, 275.195] |
| Smith and Krajbich (2018) Food | 4.464 [-2.178, 11.019] | -1.761 [-7.776 , 4.262] |
| Stewart et al. (2016) | -12.814 [-198.123 , 179.602] | -75.78 [-234.172, 79.162] |

Note. Results are displayed separately for the GLM in which the boundary separation parameter was not controlled for, and the GLM in which the boundary separation parameter was controlled for. Boldface indicates credible effects.

C.3. Analysis of choice problems with equal-valued options

The empirical analyses presented in the main text did not cover behavior in choice problems in which the options had equal values. In such choice problems, accuracy cannot be evaluated. Moreover, reward rate is merely a function of response time, not of the chosen option (since the value of the chosen option is constant within each problem). For completeness, it was nevertheless tested whether stronger attentional biases were linked to a reduction in response time and an increase in reward rate in choice problems offering equal-valued options.

The analyses included a total of 2,786 responses from 177 participants. The experiment by Stewart et al. (2016) and Experiment 2 by Fiedler and Glöckner (2012) did not include choices between equal-valued options.⁹ To quantify the severity of attentional bias, the options within each choice problem were labeled option 0 and option 1 (instead of the nonapplicable labels "higher-valued option" and "lower-valued option"). In each data set and in each trial, the absolute dwell time on option 1, *abs. dwell*₁, and option 0, *abs. dwell*₀, were calculated. The absolute dwell time on any option is given by *abs. dwell*₁ + *abs. dwell*₀ = *abs. dwell*₁, and option of time fixating on option 1 is calculated as *prop. dwell*₁ = *abs. dwell*₁ / *abs. dwell*₁ + *abs. dwell*₀ = *abs. dwell*_{any}. The proportion of time fixating on option 1 is calculated as *prop. dwell*₁ = *abs. dwell*₁ / *abs. dwell*_{any}. The severity of attentional bias is given by *att. bias* = *abs*(*prop. dwell*₁ - 0.5). The measure *att. bias* ranges from 0 to 0.5, where 0 indicates no attentional bias (i.e., both options are attended to for equal amounts of time) and 0.5 indicates an extreme attentional bias to one of the options. The data from trials with equal-valued options were analyzed using the same statistical approach also used for the data from trials with options that differed in value, with the exception of accuracy. The models were only adjusted in one regard: Due to the relatively small number of trials offering a choice between equal-valued options within each participant, the models including random effects for participants could not be reliably estimated. Therefore the participant-level random effects were removed from the models.

Table C2

Coefficients and 95% posterior intervals for the GLMs for effects of the severity of attentional bias on response time and reward rate, for choice problems offering equal-valued options.

| Dependent variable | Study | Coefficient 95% PI |
|--------------------------|---|--------------------------------|
| RT | Krajbich et al. (2010) | -0.355 [-0.853, 0.147] |
| | Smith and Krajbich (2018) Food risk | -1.708 [-2.005 , -1.401] |
| | Smith and Krajbich (2018) Monetary risk | -2.413 [$-3.62, -1.241$] |
| | Smith and Krajbich (2018) Food | -0.624 [-0.779 , -0.469] |
| | Fiedler and Glöckner (2012) Exp. 1 | -0.803 [-1.651 , 0.05] |
| Reward rate | Krajbich et al. (2010) | 1.636 [0.169, 3.118] |
| | Smith and Krajbich (2018) Food risk | 7.003 [5.846, 8.155] |
| | Smith and Krajbich (2018) Monetary risk | 181.721 [106.756, 260.327] |
| (continued on next page) | | |

⁹ Table A3 from Fiedler and Glöckner (2012) states that Experiment 2 included one choice problem with equal-valued options. This holds if the expected values are rounded to the second digit. However, the nonrounded expected values (which were used in the current analyses) differ on all choice problems.

| Table C2 (continued) | | |
|----------------------|------------------------------------|-----------------------|
| Dependent variable | Study | Coefficient 95% PI |
| | Smith and Krajbich (2018) Food | 3.011 [1.887, 4.149] |
| | Fiedler and Glöckner (2012) Exp. 1 | 1.119 [-0.025, 2.205] |

Note. Boldface indicates credible effects.

Results from all GLM analyses are displayed in Table C2. More severe attentional biases were credibly linked to a reduction in response time in the data from all tasks by Smith and Krajbich (2018), but not in Experiment 1 from Fiedler and Glöckner (2012) or in the data from Krajbich et al. (2010). In these data sets the coefficient was negative but the effect was not credible. More severe attentional biases were credibly linked to an increase in reward rate in data from all tasks by Smith and Krajbich (2018) and Krajbich et al. (2010), but not in Experiment 1 from Fiedler and Glöckner (2012)

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